Towards a Theoretical Explanation of Time-Varying Trading

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Abstract

Investors do not continuously modify their portfolio allocations with the arrival of new information. Rather, extended periods of inertia are alternated with brief moments of action where asset allocations are updated according to the current state of the economy. We explain this pattern of behavior by developing a theoretical framework that introduces information costs into a continuous-time model of asset allocation with time-varying investment opportunities. Our model explains why individuals' attention to their investment portfolio is high when the expected gains from investing in the stock market are large, such as during recessions. It also explains why investors' portfolio adjustments are lumpy in nature. The model implies a weak and time-varying co-variation between consumption growth and equity returns and so helps explain why the equity premium can be high despite the low correlation between stock returns and consumption changes. The microeconomic implications of our analysis are tested empirically against competing models of asset allocation using duration models estimated on Odean (1999)'s brokerage account data and through panel data estimates based on the PSID. Our macroeconomic predictions are supported by tests based on both high and low frequency economic and financial time-series.

Keywords: Portfolio Allocation, Sharpe Ratio, Information Costs, PSID, Ornstein-Uhlenbeck process.

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1 Introduction

Most theoretical models of asset allocation envision a frictionless environment where consumption and investment decisions are made by agents who continuously exploit all the available information. Yet, a pervasive finding in empirical finance is that investors revise their portfolio allocations and consumption decisions rather infrequently. For example, using no-load mutual fund data, Johnson (2010) estimates that investors trade once every 20 months on average. Recent research also shows that financial agents do not trade uniformly through time, implying that investors’ attention to the stock market is time-varying. This gap between theory and empirical findings is the motivation of our work.

This paper introduces information costs into a continuous-time asset pricing framework with time-varying investment opportunities and proposes a model that contributes to the financial economics literature by explaining investors’ time-varying trading and inertia in their asset allocation. When the cost of inattention is low as in those states of the world characterized by low expected returns on the stock market, investors choose to trade as infrequently as once every two or three years. Conversely, when the expected gains from investing in the stock market are high, investors pay close attention to the stock market and place numerous trades within a given month. Time-varying attention to the stock market implies a co-variation between returns and consumption that is generally weak, but increases sharply during market booms and crashes. In this respect our model contributes also to the macro-finance literature that provides a theoretical explanation for the equity premium puzzle.

We formally characterize the optimal inattentive behavior of a representative agent who pays information costs on his investment portfolio. The cost is proportional to the contemporaneous value of the portfolio and is intended to summarize the financial and non-financial costs associated with processing the information needed to formulate the optimal investment and consumption plans. Once information costs are introduced, it is no longer optimal for the investor to make consumption and portfolio decisions on a continuous basis and periods of inattention to the stock market become optimal. Our analysis shows that the length of optimal inattention depends on the economic conditions faced by the agent at the time of action: if the agent expects his investment portfolio to be very profitable, he will pay close attention to the stock-market; he will instead disregard it for a long time if expected returns are low. The mechanism is generated by a cash-in-advance constraint that forces the investor to store part of his wealth in a low-yield account (a transactions account) to finance his every-day consumption. When expected returns on the market are high, such as during recessions, the agent has the incentive to keep very little of his wealth in the transactions account and frequently transfer wealth from the investment to the transactions account. The opposite holds true if expected returns on the market portfolio are
Recognizing explicitly that the agent’s optimal behavior depends on the economic conditions he faces at the time of action is one of the key insights of our model. Conversely, by ignoring time-variations in investment opportunities, extant partial equilibrium analyses in this literature are essentially static: the agent always faces the same economic conditions, always makes the same portfolio and consumption decisions and chooses the same optimal inattention interval.

The theoretical contributions of our paper can be summarized as follows. First, consistent with the results in Abel, Eberly, and Panageas (2007, 2009) as well as Gabaix and Laibson (2002), our analysis shows that small information costs generate lengthy periods of inattention. Second, we show that the period of optimal inattention is not fixed but depends on the prevailing risk-return trade-off as summarized by the market Sharpe ratio. The value of the Sharpe ratio at the time of action determines the optimal portfolio allocation, the expected return on the investment portfolio and consequently the amount of wealth in the transactions account. Third, in our model the co-variation between returns and consumption is weak and time-varying. Because the correlation between aggregate consumption and market returns is generally low, but increases sharply during market booms and crashes, our model entails that standard CCAPM estimates are affected by a time-varying bias. Fourth, the inattention periods followed by sudden awakenings give rise to lumpy portfolio adjustments, a feature that theoretical models have difficulties in explaining. Finally, financial economists have explored exact solutions to optimal non-myopic strategic asset allocation for agents with different investment horizons, but the investment horizon has always been taken as an exogenous parameter.1 In our model the horizon is endogenously determined by the trade-off between the opportunity cost of keeping wealth in the transactions account and the cost incurred every time the value of the portfolio is observed and funds transferred from the investment to the transactions account.

Allowing for a multi-agent extension of the standard framework implies a number of additional results. First, our model explains the cross-sectional dispersion of portfolio allocations between the agents populating the economy. Identical agents who differ only in their first day of trading are shown to hold different portfolio allocations over time. Furthermore, the dispersion of portfolio allocations across agents is shown to be time-varying and positively related to the absolute value of the conditional Sharpe ratio. Second, even if all agents are endowed with the same wealth on their first trading day, different investment decisions translate into different wealth processes for different agents so that over a given investment horizon, some see their wealth increase dramatically while others end up poorer. Finally, the model implies that the aggregate trading behavior is time-varying and is a function of the Sharpe ratio of the aggregate market.

Introducing time-variations in the investment opportunity set is important in light of the large

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literature assessing the predictability of aggregate stock returns and market volatility. Numerous studies, see Lettau and Ludvigson (2009) for a comprehensive treatment, show that excess stock market returns can be predicted using financial and economic time-series. The same holds for volatility, that has been shown to be forecastable at horizons ranging from one day to six years. While classic asset pricing models such as the CAPM imply a constant price of risk, the conditional Sharpe ratio, which is defined as the ratio of conditional excess return and volatility, has been shown empirically to be strongly countercyclical and highly volatile. Our modeling strategy implies that the testable implications we derive depend only on the conditional Sharpe ratio, as the latter is the only state variable of our model. Once estimates of this quantity are obtained, empirical tests can easily be computed. We conduct four distinct empirical exercises that estimate quantitatively the micro and macroeconomic predictions of our framework.

The first tests whether the conditional Sharpe ratio affects individuals’ trading frequencies using a duration model estimated on Odean (1999)’s brokerage account data. Our findings indicate that time-varying investment opportunities are a fundamental determinant of time-varying trading and that a one-standard deviation increase in the conditional Sharpe ratio leads to a 20% increase in households’ trading probabilities. Our work is the first to analyze the impact of the investment opportunity set on households’ trading. While the number of empirical studies in the field of household finance has grown rapidly in the recent past, the majority are concerned with assessing behavioral biases in investors’ trading. Odean (1998) uses account-level data to assess whether investors are reluctant to realize losses and Odean (1999) employs a similar dataset to study whether investors’ trade too much. Huberman and Jiang (2006) analyze whether the structure of 401(k) plans influence investors’ positions in risky assets. A number of papers analyze the socio-economic determinants of individuals’ portfolio allocation and stock market participation. Ameriks and Zeldes (2002) analyze how equity positions in investors’ portfolios vary with age. Vissing-Jorgensen (2002) proposes a model explaining the non-participation puzzle, i.e. the empirical finding that a large fraction of US households do not participate in the stock market, and tests it using data from the PSID. The analyses closest to ours are the ones undertaken in Johnson (2004) and Johnson (2010). The first uses a proprietary panel dataset in one no-load mutual fund family to show that observable shareholder characteristics can be exploited to determine their investment horizon. Using the same dataset, the second shows that mutual fund shareholders do not trade uniformly through time and that while demographic traits are important trade predictors, time effects are much more important.

To estimate the extent to which the conditional Sharpe ratio affects investors’ positions in risky assets our second empirical exercise uses panel data estimates from the PSID. Our findings show that changes in investment opportunities are important determinants of investors’ portfolio
allocations: a 10% change in the average conditional Sharpe ratio increases by approximately 4% the proportion of risky assets held by investors. Our analysis nests that of Brunnermeier and Nagel (2008) who use the same dataset to show that wealth fluctuations are not associated with changes in investors’ risky asset positions, as habit-persistence models predict.

Turning to the model’s implications for the macroeconomy, we test the time-varying covariance between stock returns and aggregate consumption using a number of macroeconomic and financial time-series. After obtaining a high frequency proxy for consumption using the Arouba-Diebold-Scotti Index of economic activity, we compute the realized covariance between consumption and stock returns at the monthly frequency using daily data. We then show that the conditional covariance between consumption and stock returns is time-varying and a function of the conditional Sharpe ratio.

Our fourth empirical exercise shows that aggregate volumes on the S&P 500 are a function of the investment opportunity set. Periods characterized by large conditional Sharpe ratios are also characterized by large trading volumes.

The rest of the paper is organized as follows. Section 2 briefly surveys the theoretical literature relevant to our work, presents the key components of the model and solves it. Section 3 analyzes its static and dynamic properties. In Section 4 we conduct the main empirical exercises. Section 5 concludes.

2 The Model

Standard continuous-time models (e.g. Merton (1969)) study the optimal investment and consumption decisions of an agent that has free access to unlimited information and does not face transactions costs. This setup entails that agents are able (and willing) to continuously re-optimize their decisions. While economists recognize the elegance of this framework, they are aware of its limitations at explaining investors’ actual behavior. Agents do not continuously take advantage of all available information, they change their investment positions infrequently and do not modify their consumption instantaneously after making a profitable (or unprofitable) trade in the market. To explain the divergence between standard theory and investors’ behavior, economists devised a wide array of explanations. Relevant to our paper are the ones that focus on the role played by transactions costs and agents’ limited information-processing capacity.

Papers under the first heading find their roots in Baumol (1952) and Tobin (1956) and share the common feature that agents are willing to hold on to cash as opposed to riskless bonds (that provide a higher rate of return) because only cash can be exchanged for consumption. The idea of employing “cash-in-advance” constraints became particularly popular in the real business cycle.
literature thereafter (e.g. Cooley and Hansen (1989)), but the emphasis was generally placed on the effect of such constraints on the dynamic behavior of wages, prices and capital accumulation in aggregate economies.

The second line of research has proceeded in two directions. The first, which includes Sims (2003) and Moscarini (2004), characterizes individuals' rational inattention by way of entropy, a concept developed in the field of information theory. In these models rational inattention is generally intended as agents’ decision to focus on a specific source or type of information out of the many available ones. The second direction comprises models that impose a cost of obtaining and processing information. Examples of this literature are Abel, Eberly, and Panageas (2007), Abel, Eberly, and Panageas (2009), Gabaix and Laibson (2002) and Duffie and Sun (1990). These papers require consumption to be purchased with a liquid asset, i.e. cash, and include information costs so that the consumer does not observe continuously the value of the stock market. Our research is closely related to this literature and generalizes the results obtained so far by allowing for a time-varying investment opportunity set.

In this paper we present a partial equilibrium model for a representative agent that lives in an economy with three assets: a liquid risk-less asset, an illiquid risk-less asset and an illiquid risky asset. The liquid asset will be referred to as the transactions account and can be thought of as a checking account whose rate of return is constant and known. Only the wealth stored in the transactions account can be exchanged for consumption. The illiquid assets constitute what we will refer to henceforth as the investment portfolio. Within the investment portfolio, wealth is allocated on the basis of the expected return and volatility of the risky asset as well as the (known) return on the risk-less asset. To capture parsimoniously the time-varying behavior of expected return and volatility we let the market expected Sharpe ratio follow an Ornstein-Uhlenbeck process. This characterization is motivated by the strong evidence of a significant cyclical variation in the market Sharpe ratio uncovered, among others, by Kandel and Stambaugh (1990), Whitelaw (1997) and Perez-Quiros and Timmermann (2000). It is also convenient as many aspects of the model can be solved in closed-form using standard techniques. The agent can observe the current value of his investment portfolio and the value of the conditional market Sharpe ratio by paying an information cost that is proportional to the value of the invested wealth. Furthermore, the agent can transfer assets from the investment to the transactions account and can change the composition of the investment portfolio only by paying the information costs. We will refer to the instant right after information costs are paid as the “moment of action” and the interval

\footnote{See also to Lettau and Ludvigson (2009) for a survey.}

\footnote{This assumption is imposed for analytical convenience: at the expense of losing the closed-form solutions we could model information costs in terms of utility. On the other hand, in the presence of a market for information investors could purchase the knowledge required to update their consumption and portfolio decisions from an external source and pay a monetary amount for it.}
between two moments of action as an “optimal inattention period”. We next provide a formal description of the model.

Assume a consumer with power utility and infinite horizon that maximizes

$$\mathbb{E}_t \left\{ \int_0^\infty \frac{1}{1 - \alpha e^{-\rho s}} ds \right\}$$

(1)

where $0 < \alpha \neq 1$ and $\rho > 0$.

The agent has access to three assets. A liquid riskless asset that pays a rate of return $r^L$, a non-liquid riskless bond trading at price $B$ with return $r$ and a non-liquid risky asset $P$ with time-varying expected return $\mu_t$ and volatility $\sigma_t$. The only asset that can be exchanged for consumption is the liquid riskless asset that the agent keeps in the transactions account. We assume that $0 \leq r^L < r$ to reflect the liquidity premium associated with the transactions account and to rule out arbitrage opportunities. The return processes for the riskless bond and risky asset are:

$$\frac{dB}{B} = r \, dt$$

(2)

$$\frac{dP}{P} = \mu_t \, dt + \sigma_t \, dZ$$

(3)

where $Z_t$ is a standard Brownian motion. The drift rate $\mu_t$ and volatility $\sigma_t$ of the risky asset are both diffusion processes. Define

$$M_t = \frac{\mu_t - r}{\sigma_t}$$

(4)

to be the conditional Sharpe ratio of the risky asset, which is assumed to follow an Ornstein-Uhlenbeck process

$$dM = -\lambda_M(M - \bar{M})dt + \sigma_M dZ_M,$$

(5)

where $\lambda_M$, $\sigma_M$, and $\bar{M}$ are positive constants and $Z_{M,t}$ is a second standard Brownian motion. The correlation between the asset’s return and Sharpe ratio processes is given by $E\{dZdZ_M\} = \rho_{sM} dt$. The opportunity set has three stochastic variables $\mu_t$, $\sigma_t$ and $M_t$ linked by the definition of the Sharpe ratio: $\mu_t = r + \sigma_t M_t$, but we know from Kim and Omberg (1996) that in this specific setting, the Sharpe ratio provides all the currently available information on present and future investment opportunities.

Wealth, $W_t$, is the sum of invested wealth $W_{I,t}$ and the funds in the transactions account, denoted by $X_t$: $W_t = W_{I,t} + X_t$. The agent can observe the value of the Sharpe ratio and the
investment portfolio only by paying a fraction $\theta$, $0 \leq \theta \leq 1$, of his invested wealth. Furthermore, funds can be transferred from the investment account to the transactions account (and vice-versa) only when the value of the invested portfolio is observed. However, because $r^L < r$ the agent arrives at the end of the optimal inattention period with no funds in the transactions accounts. This means that transfers will only occur from the investment account to the transactions account.\footnote{Abel, Eberly, and Panageas (2009) show that this result holds under more general specifications for information and transaction costs.} It follows that for an investor that observes the value of his invested portfolio at the time of action $t_j$, the $t_j^+$ invested wealth is simply the current wealth, minus the amount deposited in the transactions account multiplied by $(1 - \theta)$: i.e. $W_{t,j}^+ = (W_{t,j} - X_{t,j})(1 - \theta)$.

At the time of action $t_j$, the consumer chooses the timespan of optimal inattention, $\tau$, the amount of wealth to be deposited in the transactions account $X$ and the fraction $\phi$ of the investment portfolio to be held in the risky asset. The agent’s optimal behavior is the following. At the moment of action the investor observes the current Sharpe ratio. Because its evolution and its relation to the risky asset are known, the agent’s expectations regarding the return of the risky asset at any horizon incorporate all the currently available information. For any horizon the agent can calculate the optimal investment portfolio, the expected end-of-period wealth\footnote{In Abel et al (2007) and in our paper, the investment portfolio is managed by a portfolio manager that continuously rebalances the portfolio, i.e. a mutual fund, following the rule dictated by the agent. This allows for greater tractability of the problem and excludes the re-balancing motive as a reason for the agent to observe the value of his portfolio.} and therefore the expected utility from the investment portfolio. He is also able to calculate the utility deriving from the wealth stored in the transactions account. The trade-off he faces is the following: on the one hand, he would like to keep very little wealth in the transactions account, because the investment portfolio offers a higher rate of return; on the other hand, he would like to transfer funds from the investment portfolio to the transactions account as infrequently as possible to avoid paying the costs of information. The economic conditions faced at the time of action affect fundamentally this decision.

2.1 Model Solution

We solve the model by first characterizing the dynamics of consumption. We then derive the optimal portfolio allocation and the optimal proportion of wealth to be deposited in the transactions account. Finally, we obtain the optimal inattention period, $\tau^*$. Proofs of propositions and theorems are provided in the Appendix.
2.1.1 Consumption dynamics

We derive a rule for optimal consumption that holds for any inattention period \( \tau \) and any amount \( X_{t_j} \) deposited in the transactions account at time \( t_j \). Given \( X_{t_j} \) and \( \tau \), the agent chooses consumption to maximize his utility:

\[
U_{t_j}(\tau) \equiv \max_{\{c_{t+s}\}_{s=0}^\tau} \int_0^\tau \frac{1}{1-\alpha} c_{t+s}^{(1-\alpha)} e^{-\rho s} \, ds
\]

subject to the constraint \( X_{t_j}(\tau) = \int_0^\tau c_{t+s} e^{-rL_s} \, ds \). 

**Proposition 1 (Consumption Rule and Maximized Utility from Consumption):** Given the wealth deposited in the transactions account, \( X_{t_j} \), and the inattention period \( \tau \), the maximized utility of consumption is:

\[
U_{t_j}(\tau) = \frac{1}{1-\alpha} X_{t_j}^{(1-\alpha)} h(\tau)^\alpha.
\]

Over the interval of inattention, consumption evolves according to the following rule:

\[
c_{t_j+s} = c_{t_j} e^{-\frac{(rL_s - \rho)}{\alpha} s}, \quad \text{for} \quad 0 \leq s \leq \tau.
\]

Eq. 7 describes how the maximized utility of consumption from period \( t_j \) to \( t_j + \tau \) depends on the utility derived by the sum deposited \( [1/(1-\alpha)]X_{t_j}^{(1-\alpha)} \) and a scaling factor \( h(\tau)^\alpha \) that depends on the degree of risk aversion \( \alpha \), the subjective discount factor \( \rho \) and the rate of return \( rL \) paid by the liquid risk-free asset. Eq. 8 characterizes how consumption over the inattention period increases or decreases depending on the relative values of \( \rho \) and \( rL \). If the agent discounts future consumption more than the rate of interest paid by the transactions account, consumption decreases over time and vice-versa, i.e. \( c_{t_j+s} \gtrless c_{t_j} \) if \( rL \gtrless \rho \). Under power utility the risk aversion coefficient \( \alpha \) is the inverse of the elasticity of inter-temporal substitution and so determines the rate at which consumption increases or decreases over time.

To build intuition, Figure 1 plots the dynamics of consumption (Top Panel) as well as the dynamics of the wealth stored in the transactions account (Bottom Panel) between periods of inattention. Consumption changes in a lumpy fashion at the time of action and varies very little during the periods of inattention. In fact, if \( rL = \rho \) consumption is constant between the times of action. Furthermore, the behavior of consumption during the inattention periods is independent from the dynamics of invested wealth. The wealth stored in the transactions account is exhausted completely when the period of optimal inattention ends. This is optimal because the transactions account pays a lower rate of returns compared to the expected returns paid by the investment.
2.1.2 Portfolio Choice

So far we have derived a rule for consumption that holds for any wealth $X_{t,j}$ stored in the transactions account and any inattention interval. We next show how the agent calculates the optimal allocation within the investment portfolio. At $t_j$, the agent observes the value of his invested wealth $W_{t,j}$ (which coincides with his total wealth) and the value of the current Sharpe Ratio $M_{t,j}$. Conditional on these, the value function $V(W_{t,j}, M_{t,j}, \tau)$ satisfies:

$$V(W_{t,j}, M_{t,j}, \tau) = \max_{X_{t,j}, \phi} U_t(\tau) + e^{-\rho\tau} E_t \left\{ V((W_{t,j} - X_{t,j})(1 - \theta)R(t_j, t_j + \tau)) \mid M = M_{t,j} \right\}.$$  
(9)

Conjecture the solution:

$$V(W_{t,j}, M_{t,j}, \tau) = \gamma(M_{t,j}, \tau) \frac{W_{t,j}^{1-\alpha}}{1-\alpha}$$
(10)

and substitute (7) and (10) into (9) to obtain

$$\frac{1}{1-\alpha} \gamma W_{t,j}^{1-\alpha} = \max_{X_{t,j}, \phi} \frac{1}{1-\alpha} X_{t,j}^{1-\alpha} |h(\tau)|^\alpha$$
$$+ e^{-\rho\tau} \frac{1}{1-\alpha} \gamma (W_{t,j} - X_{t,j})^{1-\alpha}(1 - \theta)^{1-\alpha} E_t \left\{ [R(t_j, t_j + \tau)]^{1-\alpha} \mid M = M_{t,j} \right\}.$$ 
(11)

The proposition that follows characterizes the optimal portfolio allocation for the investment portfolio and the associated expected returns.

**Proposition 2 (Optimal Portfolio Allocation and Maximized Expected Returns):**

Given the Sharpe ratio value $M_{t,j}$ and investment horizon $\tau$, the optimal fraction of the investment portfolio in the risky asset is:

$$\phi^* = \frac{M_{t,j}}{\alpha \sigma_{t,j}} + \frac{(C(\tau)M_{t,j} + B(\tau)) \rho \sigma_M}{\alpha \sigma_{t,j}}.$$  
(12)

The associated expected returns on the investment portfolio from $t_j$ to $t_j + \tau$ can be written as:

$$E_t \left\{ [R(t_j, t_j + \tau)]^{1-\alpha} \mid M = M_{t,j} \right\} = \exp \left\{ r(1-\alpha)\tau + A(\tau) + B(\tau)M + C(\tau) \frac{M^2}{2} \mid M = M_{t,j} \right\}.$$  
(13)

It is not straightforward to compute closed form comparative statics of Eqs. 12 and 13 because the terms $A(\tau)$, $B(\tau)$ and $C(\tau)$ are rather intractable. We can however build intuition regarding the optimal portfolio allocation and expected returns by focusing on the “normalized
return process”, which is the return obtained by buying $1/\sigma_t$ of the risky asset financed by going short in the risk-free asset:

$$dR_t = M_t \, dt + dZ.$$  \hfill (14)

The normalized return process has four parameters: $\bar{M}$, $\lambda_M$, $\sigma_M$ and $\rho_{sM}$. The first three measure the Sharpe ratio’s long-run average, the strength of its mean-reversion and its volatility, respectively. The fourth is the cross-sectional correlation between the risky asset’s return and the Sharpe ratio. As shown by Kim and Omberg (1996), to analyze the optimal portfolio allocation, it suffices to understand the role of $\bar{M}$, $\rho_{sM}$ and that of the derived parameter $k^* = (\sigma_M/\lambda_M)^2 + 2\rho_{sM}(\sigma_M/\lambda_M)$.

$\bar{M}$ controls the long-run value for the Sharpe ratio. To understand the sign and magnitude of $\rho_{sM}$, imagine that the current price for the risky asset is $P_0$ and that the agent expects future prices to follow a certain trajectory over the interval $[0, T]$. Now imagine that negative news regarding the risky asset lowers the price at time 0 to $P_0'$ and shifts downward the entire price trajectory over the interval $[0, T]$. If the fall in the current price is large compared to the fall in expected future prices, the trajectory will rotate counter-clockwise, causing the current Sharpe ratio to rise. This would produce a negative correlation between the asset’s return and the Sharpe ratio, resulting in a bias towards higher expected returns after a series of price decreases and vice-versa. If small changes in the current price are associated with changes in expected future prices in the same direction, the correlation between the risky asset’s return and the Sharpe ratio will be positive, leading to a bias towards higher expected returns after price rises and vice-versa. To sum up: the magnitude of the correlation $\rho_{sM}$ depends on how much expected future prices are influenced by information uncorrelated with the current price, while the sign of $\rho_{sM}$ determines the sign of the intertemporal correlation in normalized asset returns over discrete time-periods. The derived parameter $k^*$ determines the long-run variance of normalized return process (Eq. 14) as summarized in the proposition that follows.

**Proposition 3** (Variance of the Normalized Return Process) (Kim and Omberg (1996)):

The variance of the normalized return process (Eq. 14) can be written as:

$$\text{Var}\{R(\tau)\} = \tau + \left(\frac{\sigma_M}{\lambda_M}\right)^2 \left(\tau + \frac{2e^{-\lambda_M \tau}}{\lambda_M} - \frac{e^{-2\lambda_M \tau}}{2\lambda_M} - \frac{3}{2\lambda_M}\right)$$

$$+ \left(\frac{2\rho_{sM}\sigma_M}{\lambda_M}\right) \left(\tau + \frac{e^{-\lambda_M \tau}}{\lambda_M} - \frac{1}{\lambda_M}\right).$$  \hfill (15)

---

\[6\] This is an important subtlety: while the normalized returns process $dR_t = M_t \, dt + dZ$ has unbounded variation and no serial correlation in $Z_t$, asset returns are inter-temporally correlated over discrete intervals due to the cumulative impact of past return $dZ$ on future risk-premia $M$, via the cross-sectional correlation between $dZ$ and $dM$ combined with the dynamics of $M_t$. 

10
from which it can be shown that \( \lim_{\tau \to 0} \frac{\partial \text{Var}\{R(\tau)\}}{\partial \tau} = 1 \) and \( \lim_{\tau \to \infty} = 1 + k^* \).

The variance of the normalized return process determines how appealing the risky asset is to the representative investor, therefore affecting the portfolio allocation and the optimal inattention interval. For a positive \( \rho_s M \), the long-run variance of the normalized process is positively related to \( \sigma_M \) and negatively related to \( \lambda_M \). A negative correlation \( \rho_{sM} \) is a necessary but not a sufficient condition for \( k^* < 0 \); in fact the necessary and sufficient condition is \( \rho_{sM} < 0 \) and \( \frac{\sigma_M}{\lambda_M} \in (0, -2\rho_{sM}) \). The minimum value for the normalized return process is \( k_{\text{min}}^* = -\rho_{sM}^* \geq -1 \) and is attained for the values \( \sigma_M/\lambda_M = -\rho_{sM} \). The intuition is that with \( \rho_{sM} < 0 \) and \( \sigma_M/\lambda_M > 0 \), low returns are associated with high Sharpe ratios. It follows that the variance of the return process decreases as the horizon increases as long as the variations in the Sharpe ratio are not too large.

### 2.1.3 Wealth in the Transactions Account and Optimal Inattention

Substituting Eq. 13 into 11 and differentiating the resulting expression w.r.t \( X_{tj} \) we obtain the optimal wealth deposited in the transactions account:

**Theorem 1 (Optimal Wealth Deposited in the Transactions Account):** Given the Sharpe ratio value \( M_{tj} \), wealth \( W_{tj} \) and investment horizon \( \tau \), the optimal amount of wealth deposited in the transactions account is:

\[
X_{tj}^* = \frac{K(M_{tj}, \tau)}{K(M_{tj}, \tau) + 1} W_{tj},
\]

where \( K(M_{tj}, \tau) = \chi^{-1}e^{S(M_{tj}, \tau)} - 1; \chi \equiv (1 - \theta)^{\frac{1}{2}} \) and

\[
S(M_{tj}, \tau) = \left[ \frac{1}{\alpha} \left( (\rho - r(1 - \alpha))\tau - A(\tau) - B(\tau)M - C(\tau)\frac{M^2}{2} \right) \right]_{M = M_{tj}}.
\]

Furthermore, the analytical expression for the \( \gamma(M_{tj}, \tau) \) conjectured in Eq. 10 is:

\[
\gamma(M_{tj}, \tau) = \left( \frac{1 - e^{-\omega\tau}}{1 - \chi e^{-S(M_{tj}, \tau)}} \right)^{\alpha} \omega^{-\alpha}.
\]
equivalent to solving the problem:

\[ F(M_t, \tau) = \max_{\tau} \frac{\gamma(M_t, \tau)}{1 - \alpha} \]

s.t. \[ \chi e^{-S(M_t, \tau)} < 1 \]

(18) (19)

where the constraint guarantees that consumption is positive at all times.

We solve the problem numerically because a closed form expression would be very hard, if not impossible, to obtain. It also would not be very informative. For \( \theta \) equal to 0.1 basis points\(^7\) the optimal period of inattention is 1.80 years, while it is 1.93 years for \( \theta \) equal to 1 basis point. Both values are close to the 20 months of inattention estimated empirically by Johnson (2010).

3 Analysis of the Model

In this section we present an analysis of the static and dynamic properties of the model. The results are computed for the following parameterization: \( \sigma_M = 1.06, \lambda_M = 0.99, \rho_{s_M} = 0.1, \alpha = 5; \bar{M} = 0.5; \dot{M} = 0.5, \rho = 0.01, r^L = 0.01, r = 0.02, \theta = 0.0001. \) The long-run mean of the Sharpe ratio \( \dot{M} \) and the volatility of its process \( \sigma_M \) are calibrated using S&P 500 realized Sharpe ratio estimates from 1960 to 2008. We assume that the process is strongly mean reverting and it has a small and positive correlation with the returns process. We also assume that that we are in the long-run equilibrium by imposing \( M = \bar{M}. \) The parameter values for the return on the liquid risk-free asset, the illiquid risk-free asset, the risk-aversion coefficient, the subjective discount factor and the cost of information are chosen to match those of Abel, Eberly, and Panageas (2007).

Under this baseline parameterization we find that that the optimal inattention span is approximately 1.80 years and the fraction of invested wealth in the risky asset, given the optimal horizon, is 0.47. These numbers provide the benchmark for the analysis we report next: we allow each parameter to vary while keeping the others fixed and track the optimal inattention span as well as the optimal portfolio allocation. We first analyze the effect of the conditional Sharpe ratio on the optimal inattention and optimal portfolio allocation. We then discuss the effect of information costs and agents’ characteristics.

3.1 Current Value of the Sharpe ratio

The advantage of incorporating a state variable in our framework is that we are able to characterize the agent’s optimal behavior conditional on the economic environment he faces at the time

\(^7\)For the other parameters we employ the standard values reported at the beginning of Section 3.
of action. This is one of the key contributions of our paper because it allows for an analysis of investors’ optimal behavior over the course of the business cycle.

Figure 2 plots the optimal inattention span (continuous line) and the optimal risky asset allocation (dashed line) for different values of the annualized conditional Sharpe ratio $M$. The $y$-axis on the left reports the years of optimal inattention, while the right hand side reports the fraction of wealth in the investment portfolio allocated to the risky asset. The values for the conditional Sharpe ratios reported on the x-axis are chosen to match the empirical estimates reported in Lettau and Ludvigson (2009).

In periods characterized by high expected returns on the stock market, represented by a high value of $M$, the agent’s optimal strategy is to allocate more than 100% of his invested wealth to the risky asset, deposit very little in the transactions account and observe the value of the portfolio very frequently. The same holds when expected returns on the stock market are large and negative: the agent shorts the risky asset and reaps the benefits of the negative expected returns. Interestingly, in sluggish economic environments characterized by a Sharpe ratio around zero, it is optimal for the agent to invest very little in the risky asset and opt for very long periods of inattention. The phenomenon deserves a closer inspection.

For a myopic investor, it is the magnitude of the Sharpe ratio rather than its sign that determines the attractiveness of the risky asset. But the non-myopic investor also considers the asymmetry in the dynamics of the Sharpe ratio process: i.e. the reversion to a positive long run Sharpe ratio $\bar{M}$. Very large Sharpe ratios offer two advantages: high expected returns in the short run, combined with an ultimate return to the long run Sharpe ratio through a region of high expected returns. Very negative Sharpe ratios offer the same type of short-run profitability (recall that the agent is allowed to short the risky asset), combined with the disadvantage of the ultimate return to a long-run Sharpe ratio through a region of low expected returns. The two effects counter each other but it can be shown that for a sufficiently low Sharpe ratio, there is a net advantage for even lower values.

The mechanism described above explains the “sail” shape of the optimal inattention depicted in Figure 2: for very negative values of the current Sharpe ratio, the agent shorts the risky asset and reaps the large benefits coming from the negative returns. The transactions account has a large opportunity cost, so it is optimal for the agent to deposit very little wealth in it and pay attention to the stock market very often. For the same reason the optimal inattention period is very small for large positive values of the Sharpe ratio. For Sharpe ratios small in absolute value, the low expected returns from the investment portfolio translates into a low opportunity

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8Negative conditional Sharpe ratios can arise in general equilibrium asset pricing models with time-varying covariance between consumption growth and the stochastic discount factor. See, for example, Boudoukh, Richardson, and Whitelaw (1997) and Whitelaw (2000).
cost for the transactions account and large periods of inattention.

3.2 Comparative Statics with Respect to the Remaining Parameters

We summarize next the results for the numerical comparative statics analysis for the remaining parameters.

*(Comparative Statics Results): Under the baseline parameterization of Section 3:*

a) An increase in the persistence $\lambda_M$ of the Sharpe ratio process increases the optimal fraction of invested wealth in the risky asset and decreases the optimal inattention period (Figure 3 (a)).

b) An increase in the volatility $\sigma_M$ of the Sharpe ratio process decreases the optimal fraction of invested wealth in the risky asset and increases the optimal inattention period (Figure 3 (b)).

c) An increase in the correlation $\rho_{sM}$ between the asset return and the Sharpe ratio processes decreases the optimal fraction of invested wealth in the risky asset and decreases the optimal inattention period (Figure 3 (c)).

d) An increase in information costs $\theta$ decreases the optimal fraction of invested wealth in the risky asset and increases the optimal inattention period (Figure 3 (d)).

e) An increase in the risk-aversion coefficient "$\alpha$" or the subjective discount factor "$1 - \rho$" decreases the optimal fraction of invested wealth in the risky asset and increases the optimal inattention period (Figure 3 (e) and (f)).

In Figure 3 (a) we allow the persistence “$\lambda_M$” of the state variable to vary from 0.01 to 1 while keeping all the other parameters constant. The continuous line represents the optimal inattention period and its values are reported in years on the left hand side. The right hand side reports the optimal portfolio allocation, intended as the fraction of wealth in the investment portfolio allocated to the risky asset. From the definition of $k^*$ we know that for positive values of $\rho_{sM}$, the more persistent the state variable the lower the long-run variance of the risky asset: i.e. $\partial k^*/\partial \lambda_M = -2 \left( \frac{\sigma_M}{\lambda_M} \right) \left( \frac{\sigma_M}{\lambda_M} + \rho_{sM} \right) < 0$. This results in a more profitable investment portfolio and increases the opportunity cost of keeping funds in the transactions account. The agent opts for a lower inattention period, places less funds in the transactions account and pays more often the information cost $\theta$.

In Figure 3 (b) we report the results for different volatility levels “$\sigma_M$” of the state variable. Contrary to the persistence case, for positive values of $\rho_{sM}$ an increase in the volatility of the state variable increases the variance of the risky asset, making it less desirable: i.e. $\partial k^*/\partial \sigma_M =$
\[ \frac{2}{\lambda_M} \left( \frac{\sigma_M}{\lambda_M} + \rho_{sM} \right) > 0. \] This results in a lower opportunity cost for the transactions account. The optimal inattention increases and the wealth allocated to the risky asset decreases.

Turning to the correlation \( \rho_{sM} \), we know from the definition of \( k^* \) that given our base choice of \( \sigma_M \) and \( \lambda_M \), the long-run risk of the stock is inversely related to \( \rho_{sM} \). The intuition is the one explained in Section 2.1.2 and repeated here for convenience: the tendency for lower returns to be followed by higher Sharpe ratios lowers the overall risk of the stock as the horizon increases. The strong positive relation between the expected return on the investment portfolio and the investment horizon motivates the optimal choice of the agent: as \( \rho_{sM} \) varies from \(-1\) to \(1\), both the optimal inattention and investment in the risky asset decrease. This relation is illustrated in Figure 3 (c).

In Figure 3 (d) we evaluate the effect of information costs on the optimal inattention interval. We allow the information costs to vary from 1 to 20 basis points. The relationship between optimal inattention and information costs is positive. Greater information costs translate into a lower opportunity cost for the transactions account leading to longer optimal inattention periods. The optimal portfolio allocation is not affected directly by the information cost, it is only affected indirectly through the horizon effect: because the risk of the stock increases with the investment horizon, the weight in the risky asset decreases with it.

The agent’s preferences for risk and inter-temporal consumption are controlled by two parameters: the degree of risk-aversion \( \alpha \) and the subjective discount factor \( 1 - \rho \). Intuition dictates that \( \alpha \) has an impact on the fraction of invested wealth allocated to the risky asset as higher risk-aversion leads to more conservative investment portfolios. This is confirmed by Figure 3 (e). The effect of \( \alpha \) on the length of optimal inattention is unclear a priori as the parameter represents both the degree of risk-aversion and the inverse of the elasticity of inter-temporal substitution. The relation is essentially flat and mildly upward sloping. The relationship between the discount factor and the optimal inattention is clear-cut as depicted in Figure 3 (f). As \( 1 - \rho \) increases, the agent discounts future consumption less, implying that he consumes less in the short-run. The diminished short-run consumption implies that a given amount stored in the transactions account will finance consumption for a longer time-span, allowing for longer optimal inattention periods. The optimal portfolio allocation is only mildly negatively affected through the horizon effect.

The comparative statics analysis reported in this section allows us to get a first feel for the implications of our theoretical framework. The real appeal of our model, though, is that it allows us to explore how the time-varying investment opportunity set affects the dynamic behavior of single and multiple-agents economies. We explore these implications next.

\[^9\text{To see this, recall that } \rho_{sM} = 0.1 \text{ in our base case and use the definition of } k^*.\]
3.3 Dynamic Behavior of the Representative Agent

It is widely known that agents do not update their portfolio holdings continuously, but it is difficult to account for time-varying lumpiness in portfolio adjustments within the context of standard continuous-time models. Our model is an exception in this respect.

In this section we simulate the process for the Sharpe ratio using the parameterization specified in Section 3 and describe the portfolio allocation and trading activity for a representative investor. The top panel of Figure 4 reports the evolution of the agent’s normalized consumption and portfolio allocation. The consumption process is represented by the black continuous line and the portfolio allocation by the red dashed line. The y-axis on the left reports consumption values, while the one on the right reports the weight of the risky asset in the investment portfolio. The middle panel presents the process for the agent’s invested wealth and the bottom panel plots the simulated process for the Sharpe ratio. The vertical dotted lines in each panel represent the periods at which the agent updates his consumption and portfolio decisions. An easy way to capture the essence of Figure 4 is to think of it as the dynamic counterpart of Figure 2.

The agent enters the three year interval presented in the figure with an invested wealth of $85,000. Approximately around year 1.3, the agent ends his period of inattention. He pays the information costs and observes the value of his investment portfolio as well as the market Sharpe ratio. Given that expected returns are relatively high, the agent increases his proportion of invested wealth in the risky asset as shown in the top panel and increases his per-period consumption. The opportunity cost of the transactions account is high, so it is optimal for the agent to store relatively little wealth in it and choose a relatively short period of inattention. Over the following half year, the agent breaks his inattention seven more times and updates his portfolio allocation and consumption according to his wealth and the current investment opportunity set. Around year 1.7 the investment opportunity set worsens and it is optimal for the agent to reduce his investments in risky assets, decrease his consumption and choose a 5 months period of inattention. When the agent “wakes-up” the investment opportunities have worsened even more, so the agent chooses an even longer period of inattention of approximately one year. He then observes a rather high conditional Sharpe ratio and so trades repeatedly (approximately nine times) over the following six months. At the end of this intense trading period, the investment opportunities worsen and the investor reduces his exposure to the stock market by placing a small fraction of his invested wealth in the risky asset, storing a large portion of his wealth in the transactions account and choosing a very long period of inattention.

Changes in consumption co-vary strongly with changes in investment opportunities and wealth only during the periods characterized by intense trading, i.e. years 1.3-1.7 and years 3.3-3.7 in

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10 We normalize to 1 the average consumption of the representative agent over the sample.
Figure 4, because only in those periods the agent updates his consumption decisions to reflect the current economic environment he faces. Consumption is instead uncorrelated with the investment opportunity set over years 2.2-3.3, because the agent opts for a long period of inattention. These findings show that models of inattention that do not account for time-varying investment opportunities can be severely mis-specified in at least two respects. First, by featuring constant periods of inattention, they imply a constant bias in CCAPM estimates while, in fact, the bias is time-varying. Second, they imply that consumption is only a function of current wealth and not of the current investment opportunities and both should matter as shown by our extended model.

A unique feature of our model is that the investor acts on periods characterized by good investment opportunities only if he happens to end his inattention during one of them. For example, by observing the bottom panel of Figure 4 it is clear that the investment opportunity set over the first half of year 2 is not very different from the one that characterize the first half of year 3 and yet the investor acts very differently. He leaves untouched his investments and consumption paths in the first and he trades repeatedly in the second. The behavior of the investor at a given point in time depends on the current Sharpe ratio only if he observes it by placing a trade and paying the associated information costs.

The peculiar features described in this section sets our model apart from the ones that have been proposed so far in the literature. Standard continuous time models entail that investors’ portfolio allocations and consumption paths change continuously through time. Models that introduce transactions costs in the standard framework imply that the agent behaves in a similar fashion if similar investment opportunities recur over time. Finally, models that incorporate inattention, but do not allow for time-varying investment opportunities imply constant portfolio allocations over time as well as a constant covariance between consumption and market returns. In the empirical section we test the merits of our theoretical framework using a duration model on brokerage account data provided by Odean (1999). Our tests show that individuals’ behavior is consistent with the trading pattern entailed by our theoretical model, but inconsistent with the one entailed by the others.

Next we present the results for a multi-agent extension of our baseline model, with the intent of describing the dynamic behavior of an aggregate economy populated by a large number of inattentive consumers.

3.4 Dynamics in an Economy with Multiple Agents

In this section we extend the results obtained above by increasing the number of agents in the economy while maintaining a partial equilibrium perspective. The approach we follow is in the
same spirit of Lynch (1996) and Gabaix and Laibson (2002) and it is aimed at showing that rather different portfolio allocations and inattention intervals can coexist across different agents, if only the initial time at which they join the economy is allowed to differ.

We introduce 1000 agents in the economy, we divide the first year in 1000 equally spaced intervals and we use these to determine when each agent is allowed to observe the Sharpe ratio for the first time. The top panel of Figure 5 reports the percentage of the population trading at a given point in time. The bottom panel depicts the Sharpe ratio process.

A low value for the Sharpe ratio implies a long period of inattention for all those agents that update their portfolio decision at that time. The opposite holds true if the value of the Sharpe ratio is high. It follows that high Sharpe ratio periods are characterized by a rather intense trading activity, while little trading takes place in low Sharpe ratio periods. This is a reflection of the simple fact that while information on the current investment opportunities is always available in this economy, it is not always optimal for the agents to exploit it. Agents take advantage of it only in those periods when the gains from actively investing in the market are high. In our example intense trading takes place during the Bull market periods, i.e. approximately years 1.3-1.8, 2.2-2.4 and 3.3-3.6, while little trading takes place in low Sharpe-ratio periods, i.e. years 1.8-2.0, 2.7-3.2 and 3.7-4.0.

The top panel of Figure 6 plots the average portfolio allocation in the economy as well as the 20th and 80th percentiles of the portfolio allocations for a given period. The bottom panel plots the value of the Sharpe ratio over time.

The average exposure to risky assets in this economy is not constant, but time-varying. Between bear and bull market periods the average portfolio allocation in risky assets varies from approximately 45% to 60%, indicating that accounting for changes in investment opportunities have the potential of explaining time-variations in investors’ risky assets positions. Furthermore, because of inattention the change in the average portfolio allocation over time is not a straightforward function of the current Sharpe ratio as implied by a standard models, but a complex function of the present and past Sharpe ratios and agents’ inattention levels.

Also due to inattention, not all investors hold the same portfolio allocations at a given point in time and the dispersion in portfolio allocations is time-varying. High Sharpe ratio periods are characterized by a proportion of the population that takes advantage of the information of high expected returns, i.e. those whose optimal inattention end sometime during the high Sharpe ratio period, and a proportion of the population that doesn’t. Because only a fraction of the population reacts to the new information available, the portfolio allocation dispersion across agents is rather large. Furthermore, given that the agents that take advantage of the high expected returns have short optimal inattention periods while the value of the Sharpe ratio stays
high, but go back to long periods of inattention when the Sharpe ratio is low, the cross-sectional dispersion of portfolio allocation shrinks over low Sharpe ratio periods.

Overall, the results reported in this section show that information costs and time-varying investment opportunities result in an economy where different agents face and exploit different information sets. In this respect our model is unique, because it explains rather simply the cross-sectional dispersion in investment positions across agents that have otherwise identical risk-profiles and investment attitudes. The rest of the paper is dedicated to testing empirically the theoretical implications of the model.

4 Empirical Framework

Our theoretical model provides testable implications along a number of dimensions. The first is that investors do not trade uniformly through time and that individuals’ trading and the investment opportunity set are closely related: periods characterized by high (absolute) Sharpe ratios should be characterized by intense trading. Unlike standard continuous-time models of asset allocations, our model does not imply that agents trade continuously through time. It also does not imply that agents trade uniformly through time as implied by models of optimal inattention that do not allow for time-varying investment opportunities. Finally, unlike frameworks that account only for transactions costs, it implies that current investment opportunities should affect investors’ behavior only at times when they pay attention to the stock market. We test empirically these predictions by estimating a duration model on the very detailed brokerage account data provided by Odean (1999).11

The second testable implication of our model is that agents’ portfolio allocations vary over time as a function of the conditional Sharpe ratio. Unlike standard asset pricing models that entail agents reacting instantaneously to changes in investment opportunities, our model implies that portfolio allocations should be characterized by a considerable amount of inertia due to agents’ inattention. We test this implication using panel data estimates from the PSID.

The model carries macroeconomic testable implications as well. We have shown in Section 3 that economy-wide trading activity should be a function of the Sharpe ratio. By the same token, the covariance between consumption and returns is a function of the length of inattention among the agents populating the economy. Shorter periods of inattention imply a greater covariance between aggregate consumption and stock returns and the opposite holds true for longer periods of inattention. We test these economy-wide implications of our model using high and low frequency macroeconomic and financial time-series.

11We are grateful to Terrance Odean for providing this data.
Our theoretical framework has a number of testable implications that rest on the econometrician’s knowledge of the conditional market Sharpe ratio. This quantity is unobservable, however. To circumvent this issue, we adopt the remedy commonly employed in the literature of obtaining conditional returns and volatility series using observable economic and financial time-series and construct conditional Sharpe ratio series in a second step.

A second obstacle that makes the empirical estimates hard to implement is that in our theoretical model the agent knows the process for the Sharpe ratio, so it is enough for him to know its current value to form expectations regarding the return on the risky asset in the subsequent periods. It is hard to replicate empirically this mechanism, as the assumptions we imposed on the process for the Sharpe ratio may be too simplistic. The empirical strategy we employ to sidestep this problem lies in obtaining conditional expected returns and volatility series at various horizons and use these in our empirical tests. The next section shows how we extract conditional returns and volatility series from financial and economic variables.

4.1 Constructing Conditional Sharpe Ratio Series

In order to empirically test our framework, we need to construct conditional Sharpe ratio series from hard data, i.e. we need to construct conditional excess returns \( E_t\{r_{t+1:t+k}\} \) and volatility \( Vol_t\{r_{t+1:t+k}\} \) series at horizon \( k \) on the basis of investors’ conditioning information set at time \( t \). Depending on the empirical study at hand we allow the horizon \( k \) to range from 1 week to 12 months.

As evidenced by the returns predictability literature, a large number of conditioning variables have the potential to explain time-variations in expected returns at monthly, quarterly and yearly horizons. Welch and Goyal (2008) show that, if included in linear regression frameworks, none of the variables commonly employed in the literature can consistently outperform the prevailing mean out-of-sample at the monthly or quarterly frequency. More recently, Rossi (2010) and Rossi and Timmermann (2010) show that the same predictor variables incorporated in a new forecasting method also known as Boosted Regression Trees (BRT) provide accurate predictions in- and out-of-sample.

At the monthly frequency, we construct conditional expected returns and volatility series according to the following model specifications:

\[
E_t\{r_{t+1:t+k}\} = f_\mu(x_t|\hat{\theta}_\mu) \tag{20}
\]
\[
Vol_t\{r_{t+1:t+k}\} = f_\sigma(x_t|\hat{\theta}_\sigma), \tag{21}
\]

where \( x_t \) represents a set of publicly available predictor variables, while \( \hat{\theta}_\mu \) and \( \hat{\theta}_\sigma \) are parameter
estimates obtained via Boosted Regression Trees.\textsuperscript{12} We use BRT at the monthly frequency because of their ability to incorporate a large amount of conditioning information without over-fitting the training dataset.

Our data comprises stock returns along with a set of twelve predictor variables previously analyzed in Welch and Goyal (2008). Stock returns are tracked by the S&P 500 index and include dividends. A short T-bill rate is subtracted to obtain excess returns. The covariates from the Welch-Goyal analysis are available during 1927-2005 and we extend their sample up to the end of 2008.\textsuperscript{13} The twelve predictors are the lagged returns, the long-term returns, the volatility, the log dividend-price ratio, the log earnings-price ratio, the log dividend-earnings ratio, the three-month T-bill rate, the T-bill rate minus a three-month moving average, the yield on long term government bonds, the yield spread between BAA and AAA rated corporate bonds, the term spread and the inflation rate. Market volatility is unobserved, so we follow a large recent literature in proxying it through the square root of the realized variance obtained from daily data.

At the weekly horizon we use instead linear estimates of the form:

\[ E_t (r_{t+1:t+k}) = \beta'_\mu x_t \]  
\[ \hat{\text{Vol}}_t (r_{t+1:t+k}) = \beta'_\sigma x_t \]  

where \( x_t \) is a set of publicly available predictor variables at the weekly frequency, i.e. the dividend yield, the earnings-price ratio, the default spread and the T-bill rate, while \( \beta'_\mu \) and \( \beta'_\sigma \) vectors of parameter estimates obtained via ordinary least squares. We employ linear models for conditional estimates at the weekly horizon, because at that frequency we have very few covariates available and the benefits of using BRT would not be substantial. Furthermore, BRT generally work well when estimated on long time-series, but observations at the weekly frequency are available only for the most recent years. Armed with conditional Sharpe ratio estimates we can test the micro and macroeconomic implications of our model.

4.2 Empirical Tests Based on Micro-Data

Next we conduct two empirical exercises on micro-data. The first evaluates investors’ trading activity as a function of the investment opportunity set by estimating duration models on Odean

\textsuperscript{12}For a detailed introduction to Boosted Regression Trees and their application in financial economics, please refer to Rossi (2010) and Rossi and Timmermann (2010).

\textsuperscript{13}We are grateful to Amit Goyal and Ivo Welch for providing this data. A few variables were excluded from the analysis since they were not available up to 2008, including net equity expansion and the book-to-market ratio. We also excluded the CAY variable since this is only available quarterly since 1952.
(1999)’s brokerage account data. The second evaluates the extent to which changes in the conditional Sharpe ratios affect investors’ positions in risky assets using panel data estimates on the PSID.

4.2.1 Individuals’ trading activity and the investment opportunity set

As mentioned in Sections 3, our model implies that individuals’ trading and the investment opportunity set are closely related: periods characterized by high (absolute) conditional Sharpe ratios should also be characterized by intense trading. By taking advantage of the very detailed data of Odean (1999), we test this prediction via a duration model.

In a continuous-time model of asset allocation with no optimal inattention (and no transactions costs), agents should continuously change their portfolio allocations to incorporate new information as it becomes available. In a model that incorporates transactions costs, agents should trade only if the changes in the portfolio allocations deliver an increase in the expected portfolio returns that are large enough to outweigh the transactions costs incurred to place the trade. Finally, in a model of optimal inattention without time-varying investment opportunities, agents’ inattention should not be a function of the conditional Sharpe ratio.

For each household in our dataset, we have access to all the transactions undertaken between January 1991 and November 1996: i.e. we have detailed information on the various accounts held by the household and the transactions undertaken therein. In accordance with our theoretical model, we use the time between transactions as a natural measure of inattention to the stock market. In Odean (1999)’s dataset, the transactions are recorded at the daily frequency and this would in principle require conditional Sharpe ratio estimates at the daily frequency as well. We compute them instead at the weekly frequency, because the latter strikes a good balance between the need to capture quick changes in investment opportunities and the need to condition our estimates on a sufficient number of covariates. Our weekly conditional returns and volatility series are obtained from the following linear models:

\[ r_{t+1:t+5} = \alpha + \beta dy_t + \gamma ep_t + \delta defspr_t + \theta rfree_t + \zeta vol_{t-4:t} + \epsilon_{t+1:t+5} \]  
\[ vol_{t+1:t+5} = \alpha + \beta dy_t + \gamma ep_t + \delta defspr_t + \theta rfree_t + \zeta vol_{t-4:t} + \epsilon_{t+1:t+5} \]

where \( r_{t+1:t+5} \) are the (excess) returns on the S&P 500 index over a weekly horizon and \( vol_{t+1:t+5} \) is the realized volatility estimate for the S&P 500 index obtained from daily data.

After computing the conditional returns and volatility estimates \( \hat{r}_{t+1:t+5} \) and \( \hat{vol}_{t+1:t+5} \), we

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14 Please refer to Odean (1999) and related papers for a more precise description of the data at hand.
construct conditional Sharpe ratios

$$\tilde{Sh}_{t+1:t+5} = \frac{\tilde{r}_{t+1:t+5}}{\text{vol}_{t+1:t+5}}$$

(26)

based on observables. We employ the latter interacted with an indicator variable \(I_{\text{trade}}\) indicating whether the household placed a trade over the previous calendar year as the covariate of interest in our baseline model specification:

$$\lambda(t, x) = \lambda_0(t) \exp\{\alpha + \beta I_{\text{trade}} \times \tilde{Sh}_{t+1:t+5} + \gamma x\},$$

(27)

where \(t\) is the number of days since the last transaction and \(x\) is a vector of constant household covariates. The vector contains information on the “Investment Objective”, “Knowledge”, “Experience”, “Tax Rate”, “Income” and “Net Worth” of each household. These covariates should affect how often households trade and are included to obtain more precise estimates of the effect of the conditional Sharpe ratio on time-varying trading.

There are four investment objectives: conservative, income, growth and speculation. Individuals are allowed to choose more than one investment objective and all possible combinations appear in the dataset. In Table 1 Panel A we report the categories, together with their frequencies to give a picture of the type of investors populating the dataset. Only 5% of the investors have conservative investment objectives and approximately 70% of the households have growth, income or both as their investment objectives. There exists some cases of rather contradictory investment objectives like the “conservative and speculation” category, for example. Fortunately, the category comprises only 0.15% of the households. Furthermore, recall that households generally hold more than one investment accounts, so it is not inconceivable that different accounts serve different investment strategies, making prima facie contradictory categories more plausible.

“Knowledge” and “experience” have four categories each: none, limited, good and extensive. In Table 1 Panels B and C we show the frequency of each category. Approximately 9% of the households claim to have no financial sophistication, while 33% believe to have limited financial expertise. The “good” and “extensive” categories are chosen by 46% and 12% of the households, respectively. The frequencies for the “experience” categories are very similar. In fact “knowledge” and “experience” have a correlation coefficient of 0.8 on our sample. Because of the strong collinearity between the two measures, we only use the first in our estimates. Including that latter instead does not change our findings.

The distributions of income and net worth are, as expected, right-skewed so we log both variables before performing our estimations. Finally, households are categorized as “active traders” if they perform 48 or more commissioned trades in a year, “affluent household” if their invested
wealth exceeds $100,000, and “general brokerage” otherwise. We first estimate our duration model for the three household categories separately to assess whether households’ behavior vary substantially across household types. We then pool our estimates over the whole dataset.

Over our sample, January 1991 to November 1996, the conditional Sharpe ratio is always positive, so our model implies a positive and monotonic relation between the agent’s attention to the stock market and the Sharpe ratio. Table 2 Panel A reports the results for a Cox semi-parametric proportional hazard model estimating Eq. 27. We report the hazard ratios and the z-statistics for the conditional Sharpe ratios as well as the constant control variables described above. The results indicate that higher Sharpe ratios are associated with higher trading probabilities for the investor. The hazard ratios for the conditional Sharpe ratio in “affluent”, “general brokerage” and “active trader” households are 1.465, 1.744 and 1.228, respectively and the pooled coefficient estimate across household types is 1.628. The coefficients’ display a high degree of statistical significance as indicated by the z-statistics as well as high economic significance. For example, an estimated hazard ratio equal to 1.628 implies that a standard deviation increase in the conditional Sharpe ratio (0.25 in our sample) increases the probability of a trade by 16.7% for the average household.

Panel B repeats the estimates of Panel A without including the recent trade indicator variable: i.e. it estimates

\[ \lambda(t, x) = \lambda_0(t) \exp\{\alpha + \beta \bar{S}_t^{\lambda_{t+1:t+5}} + \gamma x\} \]  

(28)

instead of Eq. 27. We estimate the two models in an effort to distinguish between our model of inattention and alternative models of asset pricing that incorporate transactions costs, but not information costs. In our model, financial agents do not observe the Sharpe ratio unless they pay information costs. The implication is that over a period of inattention, the investor would not be able to reap the benefits from periods of high conditional Sharpe ratios because he literally would not be aware of them. In a model that incorporates transactions costs but not information costs, on the other hand, the agent would always know the value of the Sharpe ratio and act accordingly. Similar coefficient estimates across the two model specifications would rule in favor of a model with transactions costs rather than information costs. On the other hand, larger coefficient estimates for the model that includes the recent trade indicator variable would indicate that a model with information costs is a better characterization of agents’ time-varying trading behavior. The results reported in Panel B show that the conditional Sharpe ratio does not seem to have an impact on trading for “affluent households” and has a slightly negative and economically weak impact on trading for “general brokerage” households. The only household group where the conditional Sharpe ratio has a positive impact are the active traders, which is
expected, given that they are the ones most likely to track closely the stock market at all times.

Comparing Panels A and B is instructive because it shows that the conditional Sharpe ratio increases the investor’s trading probability only if the agent has been recently paying attention to the stock market. It therefore provides evidence supporting our model of information costs against those featuring only transactions costs. Models of inattention that do not allow for time-varying investment opportunities cannot be consistent with our findings, because they imply that trading probabilities are constant over time and not a function of the conditional Sharpe ratio. Finally, standard models of time-varying asset allocation cannot be consistent with our findings, because they require that agents trade continuously over time.

Turning to the constant covariates, the results show that compared to the base group that has no financial sophistication, limited knowledge of the stock market generally implies a lower probability of trading as the hazard ratios average approximately 0.9 across the three household groups. For affluent and general brokerage, good and extensive knowledge of financial markets is associated with greater trading probabilities with hazard ratios averaging approximately 1.15 and 1.35 across the two household groups. In the active traders group, instead, agents with extensive knowledge of financial markets trade as often as agents with no financial expertise, while agents with good knowledge trade significantly less. The non-monotonic relation between trading and financial expertise is likely the outcome of two competing effects. On the one hand, agents with more experience and knowledge of financial markets realize rather quickly that stock picking and market timing skills are very hard to develop and buy-and-hold strategies are often more remunerative. On the other hand, the higher the degree of financial sophistication, the greater the number of trading signals an agent is likely to receive; it is also possible that over-confidence could play a role in the active investment strategies pursued by this group.

The reason why active traders behave differently from the other two groups is probably due to the fact that the competing effects mentioned above have different relative strengths in this group. Furthermore, it is hard to believe that individuals that open an investment account claiming to place more than 48 trades a year (approximately one trade a week!) do not have any degree of financial sophistication.

Turning to the investment objectives, affluent and general brokerage accounts generally have higher trading probabilities if their investment objectives contain growth or speculation. “Income” is instead associated with less trading. The table separates the large investment objectives groups that characterize at least 1% of the sample, from the small ones. The latter groups tend to have estimated coefficients that are less significant and more erratic in general, reflecting the smaller number of observations they are estimated on. The results for active traders are slightly out of line compared to the ones displayed by the other two household groups: speculation is asso-
associated with more frequent trading, while growth and income are associated with longer duration between transactions.

Tax and income tend to have statistically and economically insignificant coefficients. As far as taxes are concerned, agents have greater incentives not to realize gains if their tax rate is high and the opposite holds true for realizing losses. Given that we do not make a distinction between buys and sales, the two effects are probably canceling each other out. Income is either insignificant or has a negative impact on trading. The coefficient is surprising at first sight, but it is probably due to the collinearity between income and net worth, which is instead significant across the board with hazard ratios equal to 1.054, 1.018 and 1.077 for affluent households, general brokerage households and active traders, respectively.

To check the robustness of our results, in Table 3 Panel A we repeat the analysis reported above by modeling parametrically the $\lambda_0(t)$ function that was estimated non-parametrically in Table 2. We model it as an exponential function because the exponential distribution is suitable for modeling data with constant (unconditional) hazard. We report only the hazard ratios and the z-statistics for the conditional Sharpe ratio and omit the results for the control variables because similar to the ones reported in Table 2.

The results are in line with the ones reported in Table 2, with the important difference that the hazard ratios are greater compared to the ones estimated using the Cox semi-parametric model. This holds true for both the specification that includes the recent trade indicator, Panel A, and the one that does not, Panel B. Economically the results are more significant. For example, a hazard ratio equal to 1.883, as estimated for the general brokerage households, implies that the a one-standard deviation increase in the conditional Sharpe ratio increases, on average, the probability of a trade by 22% if the household has placed a trade some time over the course of the previous calendar year.

In Panels B and C of Table 3 we present the coefficient estimates from a logistic regression model specification. To perform the analysis, we first construct an indicator variable denoting whether a trade occurred in a given week and convert the dataset into weekly observations. We then estimate the following two model specifications:

$$Pr(\text{trade} = 1 | \hat{Sh}_{t+1:t+5}, x) = \frac{\exp\{\alpha + \beta I(\text{trade}) \times \hat{Sh}_{t+1:t+5} + \gamma x\}}{1 + \exp\{\alpha + \beta I(\text{trade}) \times \hat{Sh}_{t+1:t+5} + \gamma x\}}$$  \hfill (29)

$$Pr(\text{trade} = 1 | \hat{Sh}_{t+1:t+5}, x) = \frac{\exp\{\alpha + \beta \hat{Sh}_{t+1:t+5} + \gamma x\}}{1 + \exp\{\alpha + \beta \hat{Sh}_{t+1:t+5} + \gamma x\}},$$  \hfill (30)

where the covariates are the same as the ones specified in the duration models described above. The results are reported in Panel B. In Panel C we estimate the above expression without the
constant covariates and introducing fixed effects in order to capture the constant households’ heterogeneity in a different fashion. In all specifications and across all household types, the coefficient for the conditional Sharpe ratio is positive and significant. Including the recent trade indicator results in more statistically and economically significant coefficients, as implied by our model of time-varying trading. Qualitatively, the logistic regression specifications with constant covariates yield similar results to the ones that employ fixed effects. Quantitatively, their coefficient estimates are smaller for the specifications that includes the recent trade indicator. They are instead uniformly larger for the specifications that does not include the recent trade indicator.

In unreported results we construct an alternative measure of time-varying investment opportunities based only on past realized Sharpe ratio estimates of the form

\[
\text{Real}_t \text{Sharpe} = \prod_{i=1}^{T} (1 + r_{t-i}) \left( \frac{\sum_{i=1}^{T} r_{t-i}^2}{T} \right)^{1/2}
\]

(31)

at the weekly, bi-weekly, monthly and bi-monthly frequencies. In the second step of our analysis we use this quantity instead of the conditional Sharpe ratio to estimate the duration and logistic regression models. The results for the realized Sharpe ratios are qualitatively consistent with the conditional ones, but smaller in magnitude.

Overall, the results reported in this section show that the changes in time-varying investment opportunities as captured parsimoniously by the conditional Sharpe ratio play an important role at explaining time-variations in investors’ trading as implied by our model. They are instead inconsistent with the models of time-varying asset pricing proposed so far in the literature.

Another important prediction of our model is that changes in investment opportunities should be associated with time-variations in investors’ exposure to risky assets. Unfortunately, the time-series dimension of Odean’s dataset is too short to test this hypothesis. We decide instead to employ panel data estimates from the PSID, because the latter dataset extends for a longer time-span even though the data is sampled at a lower frequency.

4.2.2 Investors’ positions in risky assets and the investment opportunity set

We test whether changes in the investment opportunity set affect investors’ portfolio allocations using panel data estimates from the PSID. The PSID is a longitudinal study that tracks family units and their offspring over time. The financial addenda relevant to our analysis were collected from 1984 to 1999 every 5 years. For a more precise description of the data, refer to Brunnermeier and Nagel (2008) as our analysis is based on the same dataset and data-cleaning procedure performed by them.

We define financial assets as the sum of holdings of stocks and mutual funds plus riskless
assets. The riskless assets are defined as the sum of cash-like assets and holding of bonds. The share of risky assets held by each household is computed as the sum of stocks and mutual funds, divided by the households’ total financial assets. We abstract from issues related to stock market participation and we include in our analysis only stock market participants.\textsuperscript{15} In order to analyze whether an increase/decrease in the conditional Sharpe ratio is associated with an increase/decrease in risky assets held in individuals’ portfolios, we first denote for each household the change in the proportion of risky assets over a $k$ years horizon as $\Delta_k \phi_{i,t} = \phi_{i,t} - \phi_{i,t-k}$, where $k = 5$ and use it as a regressand in a linear panel data model. Our model predicts that investors’ positions in risky assets are a function of the conditional Sharpe ratio, but also that inattention introduces inertia in investors’ portfolio weights. In order to capture this idea in a reduced form model, we use yearly conditional Sharpe ratio estimates obtained from BRT and define the average conditional Sharpe ratio over a five year horizon preceding (and including) time $t$ as a regressor and denote it by $\hat{AVG}_{t-k}^{Sh_{t-(k-1):t}}$. We use $t-k+1$ instead of $t-k$, because the conditional Sharpe ratio at time $t-k$ likely to affect the change in portfolio allocation at time $t-k$. Given that the PSID data collection is carried out throughout the year, but we do not know when each household is interviewed, we use the conditional Sharpe ratios from June of each year in our analysis. Given that our covariate of interest is the change in the average conditional Sharpe ratio, we then compute $\Delta_k \hat{AVG}_{t-(k-1):t}^{Sh_{t-(k-1):t}}$.

While the change in the conditional Sharpe ratio is the key quantity of our theoretical model, decades of research in financial economics have identified a multitude of socioeconomic variables that determine investors’ positions in risky assets. In order to account for their effects we follow Brunnermeier and Nagel (2008) and condition our estimates on a vector $q_{t-k}$ of household characteristics that includes a broad range of variables related to the life-cycle: age and $age^2$; indicators for completed high school and college education, respectively, and their interaction with $age$ and $age^2$; marital status and health status; the number of children in the household and the number of people in the household. It also includes variables related to the household’s employment and financial situation: a number of dummy variables for any unemployment in the $k$ years leading up to and including year $t-k$; the coverage of the household head’s job by a union contract; the log of the equity in the vehicles owned by the household; the log family income at time $t-k-4$; the two-year growth in log family income at $t-k$ and $t-k-2$ and a variable for inheritances received in the $k$ years leading up to and including year $t-k$.

In addition to the life-cycle variables reported above, we include preference shifters that control for changes in some household characteristics between $t-k$ and $t$ and denote them by $\Delta_k h_t$: changes in family size; changes in the number of children; sets of dummies for house

\textsuperscript{15}For analyses of the determinants of households’ stock market participation, please refer to Brunnermeier and Nagel (2008) and Vissing-Jorgensen (2002).
ownership, business ownership and non-zero labor income at \( t \) and \( t - k \). The idea to control for house ownership is motivated by the body of research showing that households might save towards buying a house by way of risky assets. When the house is purchased, the risky assets are sold, reducing dramatically the proportion of risky assets in the household’s portfolio (see Faig and Shum (2002)). In addition, we recognize that changes in the economic conditions may vary across regions, so we include the four PSID geographical regions. As a final control variable we include the change in wealth \( \Delta_k w_{i,t} \), because habit persistence models imply that changes in wealth affect risk-aversion, and hence investors’ portfolio allocations.

Our estimation equation can then be written as

\[
\Delta_k \phi_{i,t} = \alpha + \beta \Delta_k \hat{AVG}_{Sh_{t-(k-1):t}} + \gamma \Delta_k w_{i,t} + \delta q_{i,t-k} + \theta \Delta_k h_{i,t} + \epsilon_{i,t} \tag{32}
\]

We report our results in Table 4 Panel A. Column 1 reports results for Eq. 32 while column 2 introduces a number of additional control variables related to asset composition: the labor income/liquid wealth ratio interacted with age, the business wealth/liquid wealth ratio, and the housing wealth/liquid wealth ratio. For both specifications, the estimates of the \( \beta \) coefficients show that changes in the average conditional Sharpe ratio have a significantly positive effect on the proportion of risky assets held by the investors. The magnitude of the coefficient is 0.392 for the baseline specification, implying an economically significant effect: a 10% increase in the difference between the average conditional Sharpe ratio and its lag value implies a 3.9% increase in the proportion of risky assets held by investors. Furthermore, with a t-statistic equal to 2.72, the coefficient is significant at conventional statistical levels.

We also report the results for the \( \gamma \) coefficient, i.e. the one for the changes in wealth. As highlighted by Brunnermeier and Nagel (2008), this is a particularly important coefficient because habit persistence models imply a positive relation between changes in wealth and the proportion of wealth invested in risky assets. In line with the findings reported by Brunnermeier and Nagel (2008), the coefficient has a sign opposite to the expected one (negative instead of positive), providing strong evidence against the ability of habit persistence models to explain micro-data.

As a robustness check, in Panel B of Table 4 we present alternative estimates of Eq. 32 by replacing \( \Delta_k \hat{AVG}_{Sh_{t-(k-1):t}} \) with \( \Delta_k \hat{Sh}_t \), i.e. by including the change in the investment opportunity set rather than the average change in the investment opportunity set. The estimates reported in Panel B show that the change in the conditional Sharpe ratio between two survey dates is a statistically significant predictor of the changes in investors’ portfolio allocations. The coefficients are economically smaller compared to the ones for the changes in the average Sharpe ratio, but this is mainly due to the effect of averaging in Panel A. They maintain a strong statistical significance though.
Overall, the estimates reported in Panels A and B of Table 4 are consistent with our model of inattention that features time-varying investment opportunities, but not with those frameworks of inattention that imply a constant conditional Sharpe ratio. Hence, our results show that allowing for a price of risk that varies over time is important because it generates a model that explains more precisely investors’ behavior.

4.3 Empirical Tests based on macro-data

We present next the empirical tests for the macroeconomic implications of our model. We first evaluate empirically the time-varying covariance between stock returns and aggregate consumption using a high-frequency proxy for the latter, i.e. the ADS index. We then assess empirically whether aggregate time-varying trading, proxied by volumes on the S&P 500, is a function of the conditional Sharpe ratios.

4.3.1 Time-varying covariance between aggregate consumption and stock returns.

As in the previous section, we use Boosted Regression Trees and employ monthly observations to compute conditional Sharpe ratios at the one-year horizon. To test whether changes in the conditional Sharpe ratio affect the covariance between consumption and stock returns, we adopt an empirical strategy similar to the one introduced by Rossi and Timmermann (2010), which entails proxying high-frequency consumption using the ADS index. The ADS index is designed to track the business conditions of the US economy at the daily frequency, by incorporating high-and low-frequency as well as stock and flow data. It is obtained by estimating via the Kalman Filter a dynamic factor model that incorporates daily spreads between 10-year and 3-month Treasury yields, weekly initial jobless claims; monthly payroll employment, industrial production, personal income less transfer payments, manufacturing and trade sales, and quarterly real GDP.

In unreported results we show that there exists a close relation between the ADS index and total, durable and non-durable consumption with uniformly positive correlations that increase with the horizon, rising from 0.15 – 0.20 at the monthly horizon to 0.40 – 0.50 at the semi-annual and 0.50 at the annual horizon.

The advantage of using the ADS-index as opposed to consumption data is that it allows for the construction of realized co-variances between monthly stock returns and consumption using daily observations, allowing us to test very precisely how consumption and stock returns co-vary as a function of the investment opportunity set.

We compute monthly “realized covariances” between stock returns and scaled changes in the
ADS index, $\hat{\sigma}_t$, from daily observations as follows:

$$\hat{\sigma}_t = \sum_{i=1}^{N_t} \Delta ADS_{i,t} \times r_{i,t},$$  \hspace{1cm} (33)

where $\Delta ADS_{i,t}$ is the scaled change in the ADS index on day $i$ during month $t$, and $r_{i,t}$ is the corresponding stock market return. The scaled changes in the ADS index are obtained by dividing the change to the ADS Index by the standard deviation of returns times the standard deviation of changes to the index.

Once the realized covariance measure is obtained, we construct estimates of the conditional covariance

$$\hat{\sigma}_{t+1:t+k|t} = f_{\text{cov}}(x_t|\hat{\theta}_{\text{cov}}),$$  \hspace{1cm} (34)

where $x_t$ represents the same set of publicly available predictor variables we employ to construct conditional expected returns and volatilities, $\hat{\theta}_{\text{cov}}$ are estimates of the parameters obtained via Boosted Regression Trees and $k$ is a month identifier that ranges from 1 to 12.\(^\text{16}\)

In the final step, we characterize parametrically and non-parametrically the relation between the conditional covariance and the conditional Sharpe ratio by estimating the following models

$$\hat{\sigma}_{t+1:t+k|t} = \alpha + \beta \hat{Sh}_{t+1:t+12|t} + \epsilon_{t+1:t+k}$$  \hspace{1cm} (35)

$$\hat{\sigma}_{t+1:t+k|t} = f(\hat{Sh}_{t+1:t+12|t}) + \epsilon_{t:t+k}$$  \hspace{1cm} (36)

The interesting aspect of working with $\hat{Sh}_{t+1:t+12|t}$ is twofold. First, it is the same conditional Sharpe ratio measure employed in our PSID panel data estimates. Second, the conditional Sharpe ratio estimates are virtually never below -0.7, threshold below which our model would predict high attention as shown in Figure 2. It follows that, empirically, we should expect a monotonic relation between the conditional Sharpe ratio and the conditional covariance between consumption and stock returns.

We report the results for estimates of Eq. (35) in Table 5. The $\beta$ coefficient is positive and significant up to the sixth month, indicating that at short horizons the correlation between stock returns and consumption is time-varying and a function of the conditional Sharpe ratio as implied by our model. The fact that the correlation is not significantly different from zero at longer horizons is natural in our framework given that the longer the time interval employed, the greater the percentage of the population that adjusts its consumption to new levels, even during periods of long inattention.

To corroborate the results reported in Table 5, we present non-parametric estimates of the\(^\text{16}\) Refer to Rossi and Timmermann (2010) for a more detailed analysis of the conditional covariance series.
relation at hand using Boosted Regression Trees. In Figure 7 we report the results at 1, 2, 3, 6, 9 and 12 months horizons. From the plots it is clear that the relation is increasing and monotonic until the sixth month while it is effectively flat and non-monotonic at longer horizons.

Overall, the results reported in this section show that an increase in the conditional Sharpe ratio is associated with an increase in the conditional covariance between stock returns and consumption, as implied by our model. Conversely, they are inconsistent with traditional models of asset pricing or models of inattention that do not allow for time-varying investment opportunities, because they both imply a constant covariance between consumption and returns. In the next section we present our final empirical exercise testing whether economy-wide trading activity is a function of the conditional Sharpe ratio.

4.3.2 Conditional Sharpe ratio and trading volumes on the S&P 500

Our model predicts that in periods characterized by large (absolute) conditional Sharpe ratios, individuals’ opportunity cost of storing wealth in the transactions account is high. It follows that agents should trade rather frequently and, consequently, aggregate volumes on the S&P 500 should be high. The opposite is true for periods where the conditional Sharpe ratio is small in absolute terms. These periods are characterized by a rather small opportunity cost of holding wealth in the transactions account, low attention to the stock market and a feeble aggregate trading activity. We test this implication of our model using aggregate trading volumes on the S&P 500.

Trading volumes have increased dramatically from 1950 through 2008. It follows that the raw volumes series shows clear signs of non-stationarity. In order to study the effect of the conditional Sharpe ratio on trading volumes, we take the natural log transformation of the latter and fit a quadratic trend. The general increase in the trading volumes is commonly attributed to the increase in wealth and financial sophistication of economic agents as well as the technological progress of financial institutions and financial markets. Our model is silent about these, so we limit ourselves to explaining deviations of aggregate volumes from their long-run trend. We define the difference between the observed volumes and their long-run trend as the de-trended volumes series “VOL” and make it the object of our analysis. We estimate

$$VOL_{t+1} = \alpha + \beta \hat{Sh}_{t+1,t+12|t} + \gamma \hat{Sh}^2_{t+1,t+12|t} + \delta \hat{Sh}^3_{t+1,t+12|t} + \epsilon_{t+1},$$  \hspace{1cm} (37)$$

where the third-order polynomial is employed to capture the non-linearities implied by the model.
We also provide an alternative specification whereby we estimate

\[ VOL_{t+1} = f(\hat{SH}_{t+1,t+12|t}; \theta) + \epsilon_{t+1}, \]

(38)

via Boosted Regression Trees. The rationale for Eq. (s) 37 and 38 is that after controlling for technological advancements and increased stock market participation, the conditional Sharpe ratio should have an impact on agents’ attention to the stock market and, consequently, their trading activity. In Figure 8, we report the fitted values for the two specifications proposed. The dashed red line plots the fitted values for the parametric model, the solid black line the ones for the BRT model. Consistent with our theoretical model, there is a clear non-monotonic relation between aggregate trading volumes and the conditional Sharpe ratio; periods characterized by large Sharpe Ratios in absolute value are also characterized by large trading volumes. This holds irrespective of whether we employ a parametric (ordinary least squares) or non-parametric (Boosted Regression Trees) model to estimate the relation at hand.

Time-varying trading volumes are not consistent with a model of inattention that does not incorporate time-varying investment opportunities. It is also not consistent with a model that does not incorporate information costs, because the latter would entail that agents trade continuously over time. There are, however, a number of potential problems associated with testing our time-varying model of inattention using aggregate S&P 500 volumes. A common objection that could be raised is that only a small fraction of the trading volumes on the S&P 500 is generated by individuals’ trades. This objection is based on the misconception that the bulk of equity investments is held by institutional investors. In fact, as reported by Barber and Odean (2000), in 1996 47% of equity investments in the United States were held directly by households, 23% by pension funds and 14% by mutual funds. Furthermore, even if institutional investors generated larger trading volumes than individual investors, it is unlikely that their time-varying trading is a function of the conditional Sharpe ratio. It follows that institutional investors’ trading should only introduce noise in our results and hence reduce the precision of our estimates, but not affect the overall identification of our empirical strategy.

5 Conclusions

This paper presents a theoretical model of time-varying inattention to the stock market by introducing information costs into a continuous-time model of asset allocation with time-varying investment opportunities. Our model explains why individuals’ attention to their investment portfolio is high during recessions as well as during stock market booms and crashes. It also rationalizes why agents do not modify their portfolio allocations gradually with the arrival of
new information, but rather alternate extended periods of inertia with brief moments of action where asset allocations are updated according to the current state the economy. By implying a weak and time-varying co-variation between consumption growth and equity returns, our model also contributes to the literature that provides a theoretical explanation of the equity premium puzzle.

Empirical tests based on Odean (1999)’s dataset and estimates conducted on the PSID support the microeconomic predictions of our model against others that have been proposed so far in the literature. Furthermore, CCAPM-type estimates based on high and low frequency economic and financial time-series support the macroeconomic predictions of our model of a time-varying covariance between consumption and stock returns.

While this paper focuses on individuals’ portfolio allocations, the potential implications of our theoretical framework are much broader. By combining information costs and time-varying investment opportunities, the multi-agent extension of our model implies that different individuals hold different conditioning information sets at any given point in time. As a consequence, our framework has the potential to serve as the micro-foundation for the reduced form models developed by Mankiw, Reis, and Wolfers (2003) and Carroll (2003) that study heterogeneities in macroeconomic expectations. These implications are currently being explored using survey data on inflation expectations.
References


Appendix

Proof of Proposition 1: For any given amount of wealth $X_{t_j}$ deposited in the transactions account at time $t_j$ and any inattention period of length $\tau$, the agent chooses consumption to solve the following constraint optimization problem.

$$U_{t_j}(\tau) \equiv \max_{\{c_{t_j+s}\}_{s=0}^{\tau}} \int_0^\tau \frac{1}{1-\alpha} c_{t_j+s}^{(1-\alpha)} e^{-\rho s} ds \tag{A-1}$$

s.t. $X_{t_j}(\tau) = \int_0^\tau c_{t_j+s} e^{-rLs} ds, \tag{A-2}$

where Eq. A-2 is the budget constraint. The budget constraint requires the present value of consumption discounted at rate $r \frac{L}{s}$ over the inattention period $\tau$ to equal the sum deposited in the transactions account $X_{t_j}$ at $t_j$. The inter-temporal marginal rate of substitution between consumption at $t_j$ and $t_j+s$ is $(\frac{c_{t_j+s}}{c_{t_j}})^{-\alpha} e^{-\rho s}$ and the gross rate of return is $e^{(r \frac{L}{s})}$. Equating the two we obtain an expression for the evolution of consumption over the interval of inattention:

$$c_{t_j+s} = c_{t_j} e^{-\frac{\omega s}{\omega}} e^{-(\rho - (1-\alpha)r \frac{L}{s})}, \text{ for } 0 \leq s \leq \tau, \tag{A-3}$$

Substitute Eq. A-3 into Eq. A-2 to obtain:

$$X_{t_j}(\tau) = \int_0^\tau e^{-\frac{\omega s}{\omega}} e^{-(\rho - (1-\alpha)r \frac{L}{s})} ds$$

$$= c_{t_j} \int_0^\tau e^{\frac{\omega s}{\omega}} \left( \frac{\rho - (1-\alpha)r \frac{L}{s}}{\omega} \right) ds$$

$$= c_{t_j} \left[ e^{\frac{\omega \tau}{\omega}} + \frac{e^{\omega 0}}{\omega} \right]$$

$$= c_{t_j} \left[ 1 - e^{\omega \tau} \right]$$

$$= c_{t_j} h(\tau),$$

where

$$h(\tau) = \int_0^\tau e^{-\omega s} ds = \frac{1 - e^{-\omega \tau}}{\omega} \tag{A-4}$$

$$\omega = \frac{\rho - (1-\alpha)r \frac{L}{s}}{\alpha}. \tag{A-5}$$

Now substitute Eq. A-3 into Eq. A-1 and use Eq. A-4 to write the maximized utility of
consumption as:

\[
U_{ij}(\tau) = \int_{0}^{\tau} \frac{1}{1-\alpha} e^{s(1-\alpha)} e^{-\rho s} ds
\]

\[
= \int_{0}^{\tau} \frac{1}{1-\alpha} \left( t_{ij} e^{-s(\mu_t-r)} \right) (1-\alpha) e^{-\rho s} ds
\]

\[
= \int_{0}^{\tau} \frac{1}{1-\alpha} e^{(1-\alpha) s} e^{-s(\rho-r)} e^{-\rho s} ds
\]

\[
= \frac{1}{1-\alpha} e^{(1-\alpha) s} \int_{0}^{\tau} e^{-s(\rho-r)} ds
\]

\[
= \frac{1}{1-\alpha} X(1-\alpha) h(\tau)
\]

This is the form stated in Proposition 1.  

**Proof of Proposition 2:** We follow Kim and Omberg (1996) and obtain a closed form solution for the optimal expected utility and portfolio allocation. Take \( \tau \) to be the investment horizon. Denote the wealth in the investment portfolio as \( W_I \), the optimal monetary investment in the risky asset as \( y^*(W_I, M, \tau) \) and the optimal expected utility as \( J(W_I, M, \tau) \). Since \( J(W_I, M, \tau) \) is the investor’s expected utility assuming optimal investment over the remaining horizon, it satisfies the dynamic programming condition:

\[
J(W_I, M, \tau) = \max_{y(W_I, M, \tau)} \left( J - J_t dt + J_{W_I} E \left\{ dW_I \right\} + \frac{1}{2} J_{W_I} J_{W_I} E \left\{ dW_I^2 \right\} \right) + J_M E \left\{ dM \right\} + \frac{1}{2} J_{MM} E \left\{ dM^2 \right\} J_{W_I} M E \left\{ dMdW_I \right\}
\]  \hspace{1cm} (A-6)

while the condition for the optimal monetary investment in the risky asset should satisfy:

\[
y^*(W_I, M, \tau) = \left( \frac{J_{W_I}}{-J_{W_I} W_I} \right) \left( \frac{\rho_t - r}{\sigma_t^2} \right) + \left( \frac{J_{W_I} M}{-J_{W_I} W_I} \right) \left( \frac{\rho_M \sigma_M}{\sigma_t \sigma_M} \right)
\]  \hspace{1cm} (A-7)

The optimal expected utility \( J(W_I, M, \tau) \) is the solution to the partial differential equation and horizon-end condition:

\[
-J_t + J_{W_I} r W_I + \frac{1}{2} J_{W_I} W_I (y^*)^2 \sigma^2 - J_M \lambda_M (M - \bar{M}) + \frac{1}{2} J_{MM} \sigma_M^2 = 0 \]  \hspace{1cm} (A-8)

\[
J(W_I, M, 0) = U(W_I)
\]  \hspace{1cm} (A-9)
For the myopic case, it can be shown that the solution is additively-separable in $W_I$ and $M$. For the non-logarithmic case, conjecture trial solutions of the form:

\[ J(W_I, M, \tau) = \Phi(M, \tau) W_I^{1-\alpha} e^{\tau(1-\alpha)} \]  
\[ \Phi(M, \tau) = \exp \left[ A(\tau) + B(\tau) M + C(\tau) M^2 / 2 \right] \]  
\[ A(0) = B(0) = C(0) = 0 \]  
\[ \Phi(M, \tau) = \exp \left[ A(\tau) + B(\tau) M + C(\tau) M^2 / 2 \right] \]  
\[ A(0) = B(0) = C(0) = 0 \]

Note that the solution is multiplicatively separable in $W_I$ and $M$, and automatically satisfies the horizon-end condition $J(W_I, M, 0) = U(W_I)$ and the second-order condition for a maximum $J_{W_I W_I}(W_I, M, \tau) < 0$. Substituting the trial solutions in Eq. A-7 and Eq. A-8 yields the following expression for the optimal monetary investment in the risky asset:

\[ y^*(W_I, M, \tau) = \frac{W_I}{\alpha} \left( \frac{M}{\sigma} + \frac{\rho s M \sigma M C(\tau) M + \sigma^2 M^2 B(\tau)}{\sigma} \right) \]  
\[ \Phi(M, \tau) = \exp \left[ A(\tau) + B(\tau) M + C(\tau) M^2 / 2 \right] \]  
\[ A(0) = B(0) = C(0) = 0 \]

The investment in the risky asset depends only on the functions $B(\tau)$ and $C(\tau)$, while expected utility depends on $A(\tau)$ as well. The functions $C(\tau)$, $B(\tau)$ and $A(\tau)$ should be chosen to satisfy Eqs. A-11, A-12 and A-14. Substitute and collect terms to obtain the quadratic expression in $M$:

\[ \left\{ \begin{array}{c} \frac{1}{2} \sigma_M^2 C^2(\tau) - \lambda_M C(\tau) - \frac{1}{2} C'(\tau) + \frac{(1-\alpha)}{2a} \left[ 1 + \rho s_M \sigma_M C^2(\tau) + 2 \rho s_M \sigma_M C(\tau) \right] M^2 \\ \sigma_M^2 B(\tau) C(\tau) - \lambda_M B(\tau) + \lambda_M M C(\tau) - B'(\tau) + \frac{(1-\alpha)}{a} \rho s_M \sigma_M B(\tau) \left[ 1 + \rho s_M \sigma_M C(\tau) \right] \\ \frac{1}{2} \sigma_M^2 \left[ B^2(\tau) + C(\tau) \right] + \lambda_M M B(\tau) - A'(\tau) + \frac{(1-\alpha)}{2a} \rho s_M \sigma_M B^2(\tau) \end{array} \right\} M^2 \]  
\[ + \left\{ \begin{array}{c} \sigma_M^2 C^2(\tau) - \lambda_M C(\tau) - \frac{1}{2} C'(\tau) + \frac{(1-\alpha)}{2a} \left[ 1 + \rho s_M \sigma_M C^2(\tau) + 2 \rho s_M \sigma_M C(\tau) \right] M^2 \\ \sigma_M^2 B(\tau) C(\tau) - \lambda_M B(\tau) + \lambda_M M C(\tau) - B'(\tau) + \frac{(1-\alpha)}{a} \rho s_M \sigma_M B(\tau) \left[ 1 + \rho s_M \sigma_M C(\tau) \right] \\ \frac{1}{2} \sigma_M^2 \left[ B^2(\tau) + C(\tau) \right] + \lambda_M M B(\tau) - A'(\tau) + \frac{(1-\alpha)}{2a} \rho s_M \sigma_M B^2(\tau) \end{array} \right\} M 
\]  
\[ \frac{1}{2} \sigma_M^2 \left[ B^2(\tau) + C(\tau) \right] + \lambda_M M B(\tau) - A'(\tau) + \frac{(1-\alpha)}{2a} \rho s_M \sigma_M B^2(\tau) = 0. \]  

where the “primes” indicate first derivatives with respect to the horizon $\tau$. In order for the solution to hold for any Sharpe ratio $M$, the coefficients on $M^2$, $M$ and 1 must vanish. The
result is the following system of first-order non-linear ordinary differential equations:

\[
\begin{align*}
\frac{dC}{d\tau} &= c C^2(\tau) + b C(\tau) + a \\
\frac{dB}{d\tau} &= cB(\tau)C(\tau) + \frac{b}{2} B(\tau) + \lambda M \dot{C}(\tau) \\
\frac{dA}{d\tau} &= \frac{c}{2} B^2(\tau) + \frac{1}{2} \sigma_M^2 C(\tau) + \lambda M \dot{B}(\tau)
\end{align*}
\]

(A-16)  
(A-17)  
(A-18)

where

\[
\begin{align*}
a &= 1 - \frac{\alpha}{\alpha} \\
b &= 2 \left[ \left( \frac{1 - \alpha}{\alpha} \right) \rho_M \sigma_M - \lambda \right] \\
c &= \sigma_M^2 \left[ 1 + \left( \frac{1 - \alpha}{\alpha} \right) \beta_M^2 \right].
\end{align*}
\]

(A-19)  
(A-20)  
(A-21)

The first differential equation involves only \( C(\tau) \), the second \( C(\tau) \) and \( B(\tau) \) and the third \( C(\tau), B(\tau) \) and \( A(\tau) \). The first is a Riccati equation with constant coefficients and we can re-cast it as the integral equation:

\[
\int_0^\tau \frac{dC}{cC^2 + bC + a} = \tau.
\]

(A-22)

The left hand side can be determined from standard integral tables, which indicates that the solution can take 4 different forms depending on the parameters \( a, b \) and \( c \). The four solutions to the set of three functions \( C(\tau), B(\tau) \) and \( A(\tau) \) can be categorized using the discriminant

\[
q = b^2 - 4ac = 4\lambda_M^2 [1 - (\gamma - 1)k^*]
\]

(A-23)

of the quadratic equation:

\[
\frac{dC}{d\tau} = cC^2(\tau) + bC(\tau) + a = 0.
\]

(A-24)

The four solutions are:

1. \( q > 0 \): normal solution
2. \( q = 0 \): hyperbolic solution if \( b \neq 0 \) and polynomial solution if \( b = 0 \)
3. \( q < 0 \): tangent solution

The only solution that is interesting to us from an economic point of view is the “normal solution”.

41
The analytical expressions for $C(\tau)$, $B(\tau)$ and $A(\tau)$ for the normal solution case are:

\[
C(\tau) = \frac{2 \left(1 - \frac{\alpha}{\alpha} \right) (1 - e^{-\eta \tau})}{2\eta - (b + \eta)(1 - e^{-\eta \tau})} \\
B(\tau) = \frac{4 \left(1 - \frac{\alpha}{\alpha} \right) \lambda_M M(1 - e^{-\eta \tau / 2})}{\eta[2\eta - (b + \eta)(1 - e^{-\eta \tau})]} \\
A(\tau) = \left(1 - \frac{\alpha}{\alpha} \right) \frac{(2\lambda_M^2 \eta^2 + \sigma_M^2)}{\eta - \bar{b}} \tau \\
+ 4 \left(1 - \frac{\alpha}{\alpha} \right) \lambda_M^2 \eta^2 \left[(2b + \eta)e^{-\eta \tau} - 4be^{-\eta \tau / 2} + 2b - \eta \right] \\
+ \frac{2 \left(1 - \frac{\alpha}{\alpha} \right) \sigma_M^2}{\eta^2 - \bar{b}^2} \ln \left| \frac{2\eta - (b + \eta)(1 - e^{-\eta \tau})}{2\eta} \right|, \\
\]

where

\[
\eta = \sqrt{4\lambda_M^2 \left[ 1 - \left(1 - \frac{\alpha}{\alpha} \right) k^* \right]} \\
k^* = \left(\frac{\sigma_M}{\lambda_M} \right)^2 + 2\rho_{sM} \left(\frac{\sigma_M}{\lambda_M} \right) \\
\]

Proof of Proposition 3: Start from the Ornstein-Uhlenbeck process for the Sharpe ratio dynamics:

\[
dM = -\lambda_M (M - \bar{M}) + \sigma_M dZ_M, \\
\]

which has the integrated form, from $t_j$ to $t_j + \tau$, equal to:

\[
M(t_j, t_j + \tau) = \bar{M} + (M_{t_j} - \bar{M}) e^{-\lambda_M \tau} + \sigma_M \int_{t_j}^{t_j + \tau} e^{\lambda_M (\tau - u)} dZ_M(u). \\
\]

Following Karatzas and Shreve (1991), the future risk premium is normally distributed with mean, variance, auto-covariance and covariance with the risky-asset’s return equal to:

\[
E\{M(t_j, t_j + \tau)\} = \bar{M} + (M_{t_j} - \bar{M}) e^{\lambda_M \tau} \\
Var\{M(t_j, t_j + \tau)\} = \left(\frac{\sigma_M^2}{2\lambda_M} \right) (1 - e^{-2\lambda_M \tau}) \\
Cov\{M(u), M(v)\} = \left(\frac{\sigma_M^2}{2\lambda_M} \right) \left(e^{-\lambda_M |u-v|} - e^{-\lambda_M (u+v)} \right) \\
Cov\{M(u), dZ(v)\} = \left\{ \begin{array}{ll}
\rho_{sM}\sigma_M e^{-\lambda_M (u+v)} & \text{for } u < v \\
0 & \text{for } u > v
\end{array} \right. \\
\]
Recall now the expression for the normalized return process:

\[ dR_t = M_t \, dt + dZ. \]

Its expected value from \( t_j \) to \( t_j + \tau \):

\[ E\{ R(t_j, t_j + \tau) \} = \bar{M} + (M_{t_j} - \bar{M})e^{\lambda_M \tau} \]

while its variance is

\[
\begin{align*}
\text{Var}\{ R(t_j, t_j + \tau) \} &= \tau + \int_{t_j}^{t_j + \tau} \int_{t_j}^{t_j + \tau} \text{Cov}(X(u), X(v)) \, du \, dv \\
&\quad + \int_{t_j}^{t_j + \tau} \int_{t_j}^{t_j + \tau} \text{Cov}(X(u), dZ(v)) \, du
\end{align*}
\]

Using the expressions reported above we can re-write this last expression as:

\[
\begin{align*}
\text{Var}\{ R(t_j, t_j + \tau) \} &= \tau + \left( \frac{\sigma_M}{\lambda_M} \right)^2 \left( \tau + \frac{2e^{-\lambda_M \tau}}{\lambda_M} - \frac{e^{-2\lambda_M \tau}}{2\lambda_M} - \frac{3}{2\lambda_M} \right) \\
&\quad + \left( \frac{2\rho_{sM} \sigma_M}{\lambda_M} \right) \left( \tau + \frac{e^{-\lambda_M \tau}}{\lambda_M} - \frac{1}{\lambda_M} \right)
\end{align*}
\]

\[ \blacksquare \]

**Proof of Theorem 1**: Given the Sharpe ratio value \( M_{t_j} \), wealth \( W_{t_j} \) and the investment horizon \( \tau \), the value function has the following form:

\[
\begin{align*}
\frac{1}{1 - \alpha} \gamma W_{t_j}^{1-\alpha} &= \max_{X_{t_j}} \frac{1}{1 - \alpha} X_{t_j}^{1-\alpha} [h(\tau)]^{\alpha} \\
&\quad + \frac{1}{1 - \alpha} \gamma (W_{t_j} - X_{t_j})^{1-\alpha} \chi^{\alpha} e^{\left[ (r(1-\alpha) - \rho) \tau + A(\tau) + B(\tau) M + C(\tau) M^2 \right] / \lambda_M} \bigg| M = M_{t_j}
\end{align*}
\]

where

\[ \chi \equiv (1 - \theta)^{\frac{1}{\alpha}}. \]

To obtain the optimal amount of wealth deposited in the transactions account \( X_{t_j}^* \), differentiate Eq. A-25 w.r.t \( X_{t_j} \) and set it equal to zero

\[ X^*(M_{t_j}, \tau) = h(\tau) \gamma^{-\frac{1}{\alpha}} \chi^{-1} (W_{t_j} - X_{t_j}^*) e^{\left[ \frac{1}{\alpha} \left( \rho - (r(1-\alpha) - \rho - A(\tau) - B(\tau) M + C(\tau) M^2) \right) \bigg| M = M_{t_j} \right].} \]
Now define:

\[
S(M_{tj}, \tau) \equiv \left[ \frac{1}{\alpha} \left( (\rho - (1 - \alpha))\tau - A(\tau) - B(\tau)M - C(\tau)\frac{M^2}{2} \right) \right]_{M = M_{tj}}
\]  
(A-28)

\[
K(M_{tj}, \tau) \equiv h(\tau) \gamma^{-\frac{1}{\alpha}} \chi^{-1} e^{S(M_{tj}, \tau)}
\]  
(A-29)

and use Eq. A-29 to re-write Eq. A-27 as:

\[
X_{tj}^* = \frac{K(M_{tj}, \tau)}{K(M_{tj}, \tau) + 1} W_{tj}.
\]  
(A-30)

Plug this last expression into Eq. A-25 together with Eq. A-28 and solve for \(\gamma\) to obtain:

\[
\gamma(M_{tj}, \tau) = \left( \frac{K(M_{tj}, \tau)}{K(M_{tj}, \tau) + 1} \right)^{1-\alpha} \gamma + \gamma(M_{tj}, \tau) \left( \frac{1}{K(M_{tj}, \tau) + 1} \right)^{1-\alpha} \chi^{\alpha} e^{-\alpha S(M_{tj}, \tau)}. 
\]  
(A-31)

Equations (A-31) and (A-29) constitute a system of two equations and two unknowns, condition-

17 on the value of the current Sharpe ratio \(M_{tj}\). The solution is found by substituting \(K(M_{tj}, \tau)\) into \(\gamma(M_{tj}, \tau)\):

\[
\gamma = \left( \gamma^{-\frac{1}{\alpha}} h \chi^{-1} e^{S} \right)^{1-\alpha} h^{\alpha} + \frac{\gamma}{\left( 1 + \gamma^{-\frac{1}{\alpha}} h \chi^{-1} e^{S} \right)^{1-\alpha}} \chi^{\alpha} e^{-\alpha S}
\]

\[
\gamma = \left( \gamma^{-\frac{1}{\alpha}} h \chi^{-1} e^{S} \right)^{1-\alpha} h^{\alpha} + \gamma \chi^{\alpha} e^{-\alpha S}
\]

\[
\gamma = \frac{\gamma \left[ \gamma^{-\frac{1}{\alpha}} h^{1-\alpha} \chi^{-1} e^{S} e^{-\alpha S} h^{\alpha} + \chi^{\alpha} e^{-\alpha S} \right]}{\left( 1 + \gamma^{-\frac{1}{\alpha}} h \chi^{-1} e^{S} \right)^{1-\alpha}}
\]

\[
\gamma = \frac{\gamma e^{-\alpha S} \chi^{\alpha}}{\gamma^{-\frac{1}{\alpha}} h \chi^{-1} e^{S} h + 1}
\]

\[\gamma = \frac{\gamma e^{-\alpha S} \chi^{\alpha}}{\left( 1 + \gamma^{-\frac{1}{\alpha}} h \chi^{-1} e^{S} \right)^{1-\alpha}}.
\]

17We temporarily suppress the dependence on \(M_{tj}\) and \(\tau\) to ease the notation.

44
Which can be re-written as:

\[ \gamma = \frac{\gamma e^{-\alpha S} \chi^\alpha}{(1 + \gamma^{-\frac{1}{2}} h \chi^{-1} e^S)^{-\alpha}} = \frac{e^{-S\alpha} \chi^\alpha}{(1 + \gamma^{-\frac{1}{2}} h \chi^{-1} e^S)^{-\alpha}} \]

\[ 1 = \frac{e^{-S\alpha} \chi^\alpha}{(1 + \gamma^{-\frac{1}{2}} h \chi^{-1} e^S)^{-\alpha}} \]

\[ 1 = \left(1 + \gamma^{-\frac{1}{2}} h \chi^{-1} e^S\right)^{\alpha - S\alpha} \]

\[ e^{S\alpha} \chi^{-\alpha} = (1 + \gamma^{-\frac{1}{2}} h \chi^{-1} e^S)^{\alpha} \]

\[ e^S \chi^{-1} = (1 + \gamma^{-\frac{1}{2}} h \chi^{-1} e^S) \]

\[ e^S \chi^{-1} - 1 = (\gamma^{-\frac{1}{2}} h \chi^{-1} e^S) \]

\[ \left(\gamma^{-\frac{1}{2}} h \chi^{-1} e^S\right) = e^S \chi^{-1} - 1. \]

Solve the expression above for \( \gamma \) to obtain:

\[ \gamma^{-\frac{1}{\alpha}} = \frac{e^S \chi^{-1} - 1}{h \chi^{-1} e^S} \]

\[ \gamma = \left(\frac{e^S \chi^{-1} - 1}{h \chi^{-1} e^S}\right)^{-\alpha} \]

\[ \gamma = \left(\frac{h \chi^{-1} e^S}{e^S \chi^{-1} - 1}\right)^{\alpha} \]

\[ \gamma = (1 - e^{-\omega \tau})^{\alpha} \omega^{-\alpha} \left(\frac{e^S \chi^{-1}}{e^S \chi^{-1} - 1}\right)^{\alpha} \]

\[ \gamma = \left(1 - e^{-\omega \tau}\right)^{\alpha} \omega^{-\alpha} \]

Replace the expression for \( \gamma \) derived above into \( K \) to obtain:

\[ K = \gamma^{-\frac{1}{2}} h \chi^{-1} e^S \]

\[ = \left(1 - \chi e^{-S}\right) \omega (1 - e^{-\omega \tau}) \chi^{-1} e^S \]

\[ = \left(1 - \chi e^{-S}\right) \chi^{-1} e^S \]

\[ = \chi^{-1} e^S - 1. \]

Hence, the solution to the system of equations is:

\[ \gamma(M_{t_j}, \tau) = \left(\frac{1 - e^{-\omega \tau}}{1 - \chi e^{-S(M_{t_j}, \tau)}}\right)^{\alpha} \omega^{-\alpha} \]

\[ K(M_{t_j}, \tau) = \chi^{-1} e^{S(M_{t_j}, \tau)} - 1. \]

\[ \blacksquare \]
Table 1. Summary Statistics for Investors’ Knowledge and Objectives in Odean (1999)’s Dataset

Panel A. Investment Objectives and associated Frequencies

<table>
<thead>
<tr>
<th>Investment Objective(s)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth</td>
<td>55.75%</td>
</tr>
<tr>
<td>Income, Growth</td>
<td>11.94%</td>
</tr>
<tr>
<td>Conservative, Growth</td>
<td>6.99%</td>
</tr>
<tr>
<td>Income</td>
<td>4.57%</td>
</tr>
<tr>
<td>Conservative, Income, Growth</td>
<td>5.86%</td>
</tr>
<tr>
<td>Conservative</td>
<td>5.00%</td>
</tr>
<tr>
<td>Conservative, Income</td>
<td>3.56%</td>
</tr>
<tr>
<td>Growth, Speculation</td>
<td>2.74%</td>
</tr>
<tr>
<td>Speculation</td>
<td>1.32%</td>
</tr>
<tr>
<td>Conservative, Income, Growth, Speculation</td>
<td>1.17%</td>
</tr>
<tr>
<td>Income, Growth, Speculation</td>
<td>0.54%</td>
</tr>
<tr>
<td>Conservative, Growth, Speculation</td>
<td>0.25%</td>
</tr>
<tr>
<td>Conservative, Speculation</td>
<td>0.15%</td>
</tr>
<tr>
<td>Income, Speculation</td>
<td>0.13%</td>
</tr>
<tr>
<td>Conservative, Income, Speculation</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Panel B. Knowledge and associated Frequencies

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>8.85%</td>
</tr>
<tr>
<td>Limited</td>
<td>33.20%</td>
</tr>
<tr>
<td>Good</td>
<td>45.82%</td>
</tr>
<tr>
<td>Extensive</td>
<td>12.13%</td>
</tr>
</tbody>
</table>

Panel C. Experience and associated Frequencies

<table>
<thead>
<tr>
<th>Experience</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>3.74%</td>
</tr>
<tr>
<td>Limited</td>
<td>35.60%</td>
</tr>
<tr>
<td>Good</td>
<td>46.66%</td>
</tr>
<tr>
<td>Extensive</td>
<td>13.99%</td>
</tr>
</tbody>
</table>

Note: This table presents summary statistics of the knowledge, experience and investment objective variables for the investors populating the brokerage account data provided by Odean (1999). Panel A presents the frequencies for the investment objectives. Panels B and C present the frequencies for the self-reported level of knowledge and experience.
Table 2. Cox Semi-Parametric Proportional Duration Model With and Without Recent Trade Indicator Variable

### A. Specification With the Recent Trade Indicator Variable

<table>
<thead>
<tr>
<th>Time-Varying Covariates</th>
<th>Account type</th>
<th>affluent</th>
<th>general</th>
<th>active</th>
<th>total</th>
<th>affluent</th>
<th>general</th>
<th>active</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.465</td>
<td>1.744</td>
<td>1.228</td>
<td>1.628</td>
<td>0.983</td>
<td>0.935</td>
<td>1.097</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(33.16)**</td>
<td>(65.06)**</td>
<td>(25.03)**</td>
<td>(94.33)**</td>
<td>(-1.48)</td>
<td>(7.85)**</td>
<td>(11.21)**</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

### B. Specification Without the Recent Trade Indicator Variable

<table>
<thead>
<tr>
<th>Constant Covariates</th>
<th>Account type</th>
<th>affluent</th>
<th>general</th>
<th>active</th>
<th>total</th>
<th>affluent</th>
<th>general</th>
<th>active</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investors’ Knowledge</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limited</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: This table reports the results of a Cox semi-parametric duration model estimating the number of days between investors’ transactions as a function of time-varying as well as constant covariates. The time-varying covariate is the conditional Sharpe ratio estimated at the weekly frequency using observable economic and financial time-series. The constant covariates are the investors’ financial sophistication (knowledge), investment objectives, tax rate, income and net worth. For each covariate we report the hazard ratio and z-statistic. Panel A interacts the conditional Sharpe ratio with a recent trade indicator variable. Panel B does not.
Table 3. Trading Probabilities and the Investment Opportunity Set

Panel A. Parametric Exponential Duration Model

<table>
<thead>
<tr>
<th></th>
<th>Affluent</th>
<th>General</th>
<th>Active</th>
<th>Total</th>
<th>Affluent</th>
<th>General</th>
<th>Active</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hazard Ratio</td>
<td>1.578</td>
<td>1.883</td>
<td>1.265</td>
<td>1.673</td>
<td>1.118</td>
<td>1.048</td>
<td>1.170</td>
<td>1.114</td>
</tr>
<tr>
<td>z-statistic</td>
<td>(43.28)**</td>
<td>(79.16)**</td>
<td>(34.66)**</td>
<td>(110.33)**</td>
<td>(10.69)**</td>
<td>(6.01)**</td>
<td>(23.23)**</td>
<td>(23.39)**</td>
</tr>
</tbody>
</table>

Panel B. Logistic Model for Trading Probabilities With Constant Covariates

<table>
<thead>
<tr>
<th></th>
<th>Affluent</th>
<th>General</th>
<th>Active</th>
<th>Total</th>
<th>Affluent</th>
<th>General</th>
<th>Active</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.451</td>
<td>0.639</td>
<td>0.261</td>
<td>0.539</td>
<td>0.080</td>
<td>0.024</td>
<td>0.150</td>
<td>0.073</td>
</tr>
<tr>
<td>t-statistic</td>
<td>(37.33)**</td>
<td>(70.83)**</td>
<td>(26.25)**</td>
<td>(94.19)**</td>
<td>(6.77)**</td>
<td>(2.75)**</td>
<td>(15.22)**</td>
<td>(12.95)**</td>
</tr>
</tbody>
</table>

Panel C. Logistic Model for Trading Probabilities With Fixed Effects

<table>
<thead>
<tr>
<th></th>
<th>Affluent</th>
<th>General</th>
<th>Active</th>
<th>Total</th>
<th>Affluent</th>
<th>General</th>
<th>Active</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.256</td>
<td>0.321</td>
<td>0.218</td>
<td>0.272</td>
<td>0.095</td>
<td>0.038</td>
<td>0.166</td>
<td>0.093</td>
</tr>
<tr>
<td>t-statistic</td>
<td>(21.27)**</td>
<td>(35.94)**</td>
<td>(21.17)**</td>
<td>(46.19)**</td>
<td>(8.03)**</td>
<td>(4.34)**</td>
<td>(16.12)**</td>
<td>(15.93)**</td>
</tr>
</tbody>
</table>

Note: This table reports in Panel A the results of a parametric exponential duration model estimating the number of days between investors' transactions as a function of time-varying as well as constant covariates. The time-varying covariate is the conditional Sharpe ratio estimated at the weekly frequency using observable economic and financial time-series. The constant covariates, whose coefficient estimates are not reported, are the investors' financial sophistication (knowledge), investment objectives, tax rate, income and net worth. For each covariate we report the hazard ratio and z-statistic. In Panel B and C we present the results of two logistic regression specifications. The first employs the same covariates used in Panel A, while the second omits the constant covariates and introduces fixed effects at the household account level instead. In all panels we present the results for a specification that interacts the conditional Sharpe ratio with a recent trade indicator variable and one that does not.
Table 4. Evidence of time-varying portfolio allocation from PSID

Panel A. Portfolio Allocations and Changes in the Average Conditional Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th>Spec 1</th>
<th>Spec 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.392</td>
<td>0.412</td>
</tr>
<tr>
<td>S.E. $\beta$</td>
<td>0.144</td>
<td>0.146</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.014</td>
<td>-0.010</td>
</tr>
<tr>
<td>S.E. $\gamma$</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>Asset control Composition</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Preference Shifters</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Life-Cycle controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Regions Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>3.9%</td>
<td>4.2%</td>
</tr>
<tr>
<td>$N$</td>
<td>1234</td>
<td>1234</td>
</tr>
</tbody>
</table>

Panel B. Portfolio Allocations and Changes in the Conditional Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th>Spec 1</th>
<th>Spec 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.035</td>
<td>0.037</td>
</tr>
<tr>
<td>S.E. $\beta$</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.014</td>
<td>-0.010</td>
</tr>
<tr>
<td>S.E. $\gamma$</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>Asset control Composition</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Preference Shifters</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Life-Cycle controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Regions Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>3.8%</td>
<td>4.0%</td>
</tr>
<tr>
<td>$N$</td>
<td>1234</td>
<td>1234</td>
</tr>
</tbody>
</table>

Note: This table reports in Panel A the results of a linear regression model estimating the relation between changes in households’ risky assets positions over the course of five years as a function of average changes in the conditional Sharpe ratio and households’ wealth over the same time-span. A number of additional covariates that control shifts in preferences, life-cycle and regional effects are included, but their coefficients are not reported. Specification 2 differs from specification 1 in that it includes variables that control for households’ asset composition. Panel B repeats the analysis substituting the average changes in the conditional Sharpe ratio with changes in the conditional Sharpe ratio. Coefficients and standard errors are reported for the regressors of interest. Coefficients significant at 5% level are bold-faced.
### Table 5. Covariance Between Consumption and Stock Returns as a Function of the Conditional Sharpe Ratio

<table>
<thead>
<tr>
<th>Horizon (Months)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>α</strong></td>
<td>-0.04</td>
<td>0.05</td>
<td>0.27</td>
<td>0.38</td>
<td>0.47</td>
<td>1.00</td>
<td>1.36</td>
<td>2.52</td>
<td>3.33</td>
<td>3.42</td>
<td>3.77</td>
<td>3.95</td>
</tr>
<tr>
<td>S.E. α</td>
<td>0.01</td>
<td>0.05</td>
<td>0.09</td>
<td>0.12</td>
<td>0.16</td>
<td>0.21</td>
<td>0.25</td>
<td>0.29</td>
<td>0.35</td>
<td>0.39</td>
<td>0.43</td>
<td>0.45</td>
</tr>
<tr>
<td><strong>β</strong></td>
<td>0.18</td>
<td>0.38</td>
<td>0.50</td>
<td>0.67</td>
<td>1.04</td>
<td>0.78</td>
<td>0.65</td>
<td>-0.35</td>
<td>-0.72</td>
<td>-0.55</td>
<td>-0.49</td>
<td>-0.28</td>
</tr>
<tr>
<td>S.E. β</td>
<td>0.02</td>
<td>0.07</td>
<td>0.13</td>
<td>0.18</td>
<td>0.24</td>
<td>0.30</td>
<td>0.37</td>
<td>0.43</td>
<td>0.52</td>
<td>0.57</td>
<td>0.63</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: This table reports \( \alpha \) and \( \beta \) coefficients and standard errors for the linear model:

\[
\hat{\text{cov}}_{t+1:t+k|t} = \alpha + \beta \hat{S}_{t+1:t+12|t} + \epsilon_{t+1:t+k},
\]

where \( \hat{\text{cov}}_{t+1:t+k|t} \) is the conditional covariance between consumption and stock returns over “\( k \)” months and \( k \) ranges from 1 to 12. \( \hat{S}_{t+1:t+12|t} \) is the one-year conditional Sharpe ratio. Both conditional covariance and Sharpe ratio measures are obtained via Boosted Regression Trees over the sample 1960-2008. Coefficients significant at 5% level are bold-faced.
Figure 1: This figure plots the dynamics of consumption (Top Panel) as well as the dynamics of the wealth stored in the transactions account (Bottom Panel) as a function of time. Consumption changes in a lumpy fashion at the times of action (represented by vertical dotted lines) and varies very little during the periods of inattention: in fact, if $r_L = \rho$ consumption is constant between the times of action. The behavior of consumption during the periods of optimal inattention is independent from the dynamics of invested wealth and the investment opportunity set. The wealth held in the transactions account is exhausted completely when the period of optimal inattention ends. This is optimal for the agent because the transactions account pays a lower rate of returns compared to the expected returns paid by the investment portfolio.
Figure 2: This figure plots the optimal inattention span (continuous line) and the optimal risky asset allocation (dashed line) for different values of the Sharpe ratio $M$. When the expected returns are high, the agent’s optimal strategy is to allocate more than 100% of his invested wealth in the risky asset, deposit very little in the transactions account and observe the value of the investment portfolio very frequently. The same holds when the expected Sharpe ratio is very high: the agent shorts the risky asset and reaps the benefits of the negative expected returns. In sluggish economic environments characterized by a Sharpe ratio around zero, the agent’s optimal portfolio choice is to invest very little in the risky asset. This translates in a low opportunity cost for the transactions account and a very long period of optimal inattention. The x-axis tracks the variable under study and the y-axis reports the years of inattention on the left and the proportion of invested wealth in the risky asset on the right.
Figure 3: This figure plots the optimal inattention interval (continuous line) and the optimal risky asset allocation (dashed line) as a function of the persistence parameter for the Sharpe ratio process $\lambda_M$ (Panel (a)); its volatility parameter $\sigma_M$ (Panel (b)); its correlation coefficient with the risky asset $\rho_{sM}$ (Panel (c)); the investor’s information costs $\theta$; the investor’s risk-aversion coefficient $\alpha$; and the investor’s subjective discount factor $1 - \rho$. In all panels, the x-axis tracks the variable under study and the y-axis reports the years of inattention on the left and the proportion of invested wealth in the risky asset on the right.
Figure 4: This figure plots the portfolio allocation and trading activity for the representative investor. The top panel reports the evolution of the agent’s normalized consumption and portfolio allocation. The consumption process is represented by the black continuous line and the portfolio allocation by the red dashed line. The y-axis on the left reports values for the consumption, while the one on the right reports the weight of the risky asset in the investment portfolio. The middle panel presents the process for the agent’s invested wealth and the bottom panel plots the simulated process for the Sharpe ratio. The vertical lines in each of the panels represent the agent’s trading activity: every period at which the agent changes his portfolio allocation is denoted by a vertical line.
Figure 5: This figure plots the trading activity for an economy composed of 1000 agents. The top panel reports the percentage of the population trading at a given point in time. The bottom panel plots the value of the Sharpe ratio over time.
Figure 6: This figure plots the dispersion of portfolio allocations across investors for an economy composed of 1000 agents. The top panel reports average portfolio allocation as well as the 20th and 80th percentiles of the portfolio allocations for a given period. The bottom panel plots the value of the Sharpe ratio over time.
Figure 7: These figures represent non-parametric estimates of the relation between the conditional covariance between consumption and stock returns and the conditional Sharpe ratio. The estimates are obtained via Boosted Regression Trees over the sample 1960-2008 and we report results at 1, 2, 3, 6, 9 and 12 months horizons. The horizontal axis covers the sample support of the conditional Sharpe ratio, while the vertical axis tracks the change in the conditional covariance.
Figure 8: This figure plots the fitted values of the aggregate trading volume on the S&P 500 index as a function of the conditional Sharpe ratio. The conditional Sharpe ratio is estimated using Boosted Regression Trees over the sample 1960-2008. The fitted values reported are obtained either by boosted regression trees (black line) or a third-order polynomial model estimated via ordinary least squares.