Asset Markets and Monetary Policy

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Introduction

- financial market and economic activity strongly interact

- appropriate monetary policy?

- Taylor rule Taylor (1993)
  respond to inflation and output gap

  Svensson (1997, 1999)
  Woodford (2003)
• Impact of asset price movements?

• Bernanke, Gertler & Gilchrist (1999)
  net worth moves procyclically
  can magnify disturbances

• Cecchetti et al. (2000)
  beneficial effects when responding to asset prices
  exogenous bubbles or non-fundamental asset price
• Dupor (2005)
capital valuations are impacted by asset price bubbles
distortions in aggregate demand

• Beaudry & Portier (2004)
asset price bubbles resulting from asset price booms

• Christiano, Motto & Rostagno (2005)
bubbles arise due to misperceived technology shocks
Coenen & Wieland (2004)

Eggertsson & Woodford (2003)

Japanese deflationary experience:

asset prices and product prices decline
and inflation rates approach zero
• **quantitative easing**

  provide private economy with liquidity
  purchasing “bad assets”
  reviving the economy through government spending
• current crises
  triggered by subprime crisis
  global financial market meltdown
  banking crisis
  downturn in real economic activity
  partial deflation

monetary policy and asset markets:
U.S.: zero interest rates
quantitative easing
• **dynamic portfolio approach**

  reveals transmission mechanism and limitations of monetary policy

  **static:** Tobin (1969, 1980)
  Frankel (1995)

  **dynamic:** Campbell & Viceira (2002)

  constant variances
  constant expected risk premia
  constant consumption-wealth ratio
  risk neutral assumptions
Benchmark Approach

Pl. & Heath (2006)

distinction between nominal and real assets
real world martingales
general stochastic processes

⇒

interest rate rule
inflation rate rule
- interest rate rule

\[ i_t = \left( a_t - \gamma \sigma_t^2 (1 - |1 - \alpha_t|) \right)^+ \]

- \( a_t \): expected return
- \( \gamma \): risk aversion
- \( \alpha_t \): fraction of wealth invested in the equity market
- \( \sigma_t \): volatility of the equity index
• inflation rate rule

\[ \pi_t = a_t - c_t + \frac{\gamma \sigma_t^2}{2} \alpha_t (\alpha_t - 1) \]

\( c_t \) consumption rate
Asset Market Dynamics

Merton (1992)
Cochrane (2001)
Campbell & Viceira (2002)

• savings account

\[ \beta_t = \exp \left\{ \int_0^t i_s \, ds \right\} \]

\( i_t \) nominal interest rate

• risky asset

\[ dP_t = P_t \left( a_t \, dt + \sigma_t \, dz_t \right) \]

\( z_t \) standard Wiener process
\( \sigma_t > 0 \)
• consumer price index

\[ I_t = \exp \left\{ \int_0^t \pi_s \, ds \right\} \]

\( \pi_t \) inflation rate

• consumption rate

\[ c_t = c_0 \exp \left\{ \int_0^t e_s \, ds \right\} \]

\( c_0 > 0 \)

\( e_t \) growth rate of consumption rate
Budget equation

total wealth

$$dW_t = W_t ((1 - \alpha_t) i_t \, dt - c_t \, dt + \alpha_t (a_t \, dt + \sigma_t \, dz_t))$$

\(\alpha_t\) fraction in the risky asset index \(P_t\)

\(W_0 > 0\).
Maximization of aggregate consumed real wealth per unit of time:

\[
\frac{c_s W_s}{I_s}
\]

- objective

\[
\max_{\mathcal{W}} E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right)
\]

for all \(0 \leq t < s < \infty\)

real world expectation

\(U'(\cdot) > 0\) \hspace{1cm} \(U''(\cdot) < 0\)
Example:

- power utility

\[ U(x) = \frac{x^{(1-\gamma)}}{1 - \gamma} \]

\( \gamma > 0, \gamma \neq 1, \)
\( \gamma \rightarrow 1 \) logarithmic utility

no time horizon

consumption rate \( c_t \) given
Market Dynamics

- **martingale approach**
  - assumes risk neutral martingales
  - Cox & Huang (1989)
  - Campbell & Viceira (2002)
  - trends in real world dynamics are ignored

- **benchmark approach**
  - assumes real world martingales
  - Pl. & Heath (2006)
  - no equivalent risk neutral probability measure needed
  - trends in real world dynamics are taken into account
Figure 1: Radon-Nikodym derivative and total mass of putative risk neutral measure.
Optimal Wealth Dynamics

- benchmark as best performing strictly positive portfolio

numeraire portfolio $S^*_t$

$$dS^*_t = S^*_t (i_t dt + \theta_t (\theta_t dt + dz_t))$$

market price of risk

$$\theta_t = \frac{a_t - i_t}{\sigma_t}$$


growth optimal portfolio
Kelly (1956), Merton (1973)
stochastic discount factor

\[ M_t = \frac{1}{S_t^*} \]

similar as Cochrane (2001) but more general

\[ dM_t = -\theta_t \, M_t \, dz_t - i_t \, M_t \, dt \]

\[ M_0 = 1 \]
• accumulated total wealth $W_t G_t$

$$G_t = \exp \left\{ \int_0^t c_s \, ds \right\}$$

benchmarked accumulated total wealth

$$\tilde{W}_t = \frac{W_t G_t}{S^*_t}$$

$$d\tilde{W}_t = \tilde{W}_t (\alpha_t \sigma_t - \theta_t) \, dz_t$$

local martingale, supermartingale

Pl. & Heath (2006)
• Law of the minimal price

Pl. (2008)

⇒

real world pricing formula

\[ U_t = S_t^* E_t \left( \frac{H_s}{S_s^*} \right) \]
• assume martingale property

\[ E_t(W_s M_s G_s) = M_t W_t G_t \]

for all \( 0 \leq t \leq s < \infty \)

martingale is most cost efficient

nonnegative supermartingale
constraint optimization problem

\[ v_t = \max_{\mathcal{W}} E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right) - \ell_t E_t \left( \tilde{W}_s - \tilde{W}_t \right) \]

\[ = \max_{\mathcal{W}} E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right) - \ell_t \left( \frac{W_s G_s}{S^*_s} - \frac{W_t G_t}{S^*_t} \right) \]

for \( 0 \leq t < s < \infty \)

\( \ell_t \) Lagrange multiplier

\( \tilde{W}_t \) martingale

only real world probability used
• candidate for optimal wealth process

\[
\left[ U\left( \frac{c_s W_s}{I_s} \right) - \ell_t \left( \frac{W_s G_s}{S_s^*} - \frac{W_t G_t}{S_t^*} \right) \right] \rightarrow \max
\]

\[
U'\left( \frac{c_s W_s}{I_s} \right) \frac{c_s}{I_s} - \ell_t \frac{G_s}{S_s^*} = 0
\]

\[
\frac{c_s W_s}{I_s} = U'^{-1} \left( U' \left( \frac{c_s W_s}{I_s} \right) \right) = U'^{-1} \left( \ell_t \frac{G_s}{S_s^*} \frac{I_s}{c_s} \right)
\]
\[ W_s = \frac{I_s}{c_s} U'^{-1}(\ell_t \phi_s) \]

with

\[ \phi_s = \frac{G_s I_s}{S_s^* c_s} \]

Lagrange multiplier \( \ell_t = \ell_0 = c_0 U'(c_0 W_0) \)

\[ 0 \leq t < s < \infty \]

generally \( W \) maximizes

\[ E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right) \]
• benchmarked accumulated total wealth

\[ \frac{W_t G_t}{S^*_t} = M_t W_t G_t = F(\phi_t) \]

with

\[ F(\phi) = \phi U'^{-1}(\ell_0 \phi) \]

\[ d\phi_t = \phi_t ([c_t + \pi_t - e_t - i_t] dt - \theta_t dz_t) \]
Itô formula

\[ d\tilde{W}_t = \frac{\partial}{\partial \phi} F(\phi_t) \phi_t ([c_t + \pi_t - e_t - i_t] \, dt - \theta_t \, dz_t) \]

\[ + \frac{1}{2} \frac{\partial^2}{\partial \phi^2} F(\phi_t) \phi_t^2 \theta_t^2 \, dt \]

Comparison of the drift coefficients

\[ 0 = c_t + \pi_t - e_t - i_t - \frac{\theta_t^2}{2\gamma} \]
Example:

power utility

\[
U'^{-1}(y) = y^{-\frac{1}{\gamma}}
\]

\[
F(\phi_s) = \ell_0^{\frac{1}{\gamma}} \phi_s^{1-\frac{1}{\gamma}}
\]

⇒

\[\gamma_t = \gamma\]
comparison of diffusion coefficients \[ \Rightarrow \]

\[ \tilde{W}_t(\alpha_t \sigma_t - \theta_t) = -\frac{\partial}{\partial \phi} F(\phi_t) \phi_t \theta_t \]

\[ \Rightarrow \]

\[ \alpha_t = \frac{\theta_t}{\tilde{\gamma}_t \sigma_t} \]

with

\[ \frac{1}{\tilde{\gamma}_t} = 1 - \phi_t \frac{\frac{\partial}{\partial \phi} F(\phi_t)}{F(\phi_t)} \]

Example: power utility

\[ \Rightarrow \tilde{\gamma}_t = \gamma \]
Optimal Interest Rate

assume power utility

⇒ optimal interest rate:

\[ \tilde{i}_t = a_t - \gamma \alpha_t \sigma_t^2 \]

cannot become negative \[ \implies \]

adjusted interest rate

\[ i_t = (\hat{i}_t)^+ \]

\( \hat{i}_t \) - theoretical interest rate

- **negative optimal interest rates**

  Japanese stagnation
great depression
current financial crisis
“quantitative easing”
• change in consumption rate

\[
\frac{dc_t}{dt} = c_t \, e_t
\]

\[
= c_t \left( c_t + \pi_t - i_t - \frac{(a_t - i_t)^2}{2\gamma \sigma_t^2} \right)
\]

\[
= c_t \left( \pi_t - \tilde{\pi}_t - \frac{(i_t - \phi_t)^2}{2\gamma \sigma_t^2} \right)
\]

with critical inflation rate

\[
\tilde{\pi}_t = a_t - c_t - \frac{\gamma \sigma_t^2}{2}
\]

and intrinsic interest rate

\[
\phi_t = a_t - \gamma \sigma_t^2
\]
Figure 2: Inflation versus interest.
applying optimal interest rate

\[ \frac{dc_t}{dt} = c_t \left( \pi_t - \tilde{\pi}_t - \frac{\gamma \sigma_t^2}{2} (1 - \alpha_t)^2 \right) \]

\[ \Rightarrow \quad \text{Inflation Rate Rule:} \]

\[ \pi_t = \tilde{\pi}_t + \frac{\gamma \sigma_t^2}{2} (1 - \alpha_t)^2 \]
1. **Subcritical Inflation**

\[ \pi_t < \tilde{\pi}_t \]

\[ \implies c_t \text{ decreases} \]

Interest rate does not matter!

Nothing can stop downward trend!

\( a_t \) may decrease

\[ \implies \text{recession} \]

**Subcritical inflation dangerous!**

\[ \implies \text{Low inflation rate targeting questionable!} \]
2. **Supercritical Inflation**

\[ \pi_t \geq \tilde{\pi}_t \]

- **without sufficient credit clearing**
  \[
  \pi_t - \tilde{\pi}_t < \frac{\gamma \sigma_t^2}{2} (1 - \alpha_t)^2
  \]
  \[\implies c_t \text{ decreasing}
  \]
  not as strong as for \( \pi_t < \tilde{\pi}_t \)

Increased borrowing may further decrease \( c_t \)!

- **reversal:**
  Making inflation sufficiently high
  and balancing credit market!
• Approximating Optimal Interest Rate

Set interest rate $i_t$ slightly above or below some interest rate

$$\bar{i}_t = \phi_t + \sqrt{2 \gamma \sigma_t^2 (\pi_t - \tilde{\pi}_t)}$$

• $i_t > \bar{i}_t$

$$\Rightarrow (i_t - \phi_t)^2 \geq 2 \gamma \sigma_t^2 (\pi_t - \tilde{\pi}_t)$$

$$\Rightarrow c_t \text{ decreasing}$$

• $\phi_t < i_t < \bar{i}_t$

$$\Rightarrow c_t \text{ increasing}$$

convenient mechanism
• for $\alpha_t < 1$ and inflation rate rule

$\implies \hat{i}_t = \tilde{i}_t$ optimal interest rate

and

$$\pi_t - \tilde{\pi}_t = \frac{\gamma \sigma_t^2}{2} (\alpha_t - 1)^2$$

• For $\alpha_t > 1$

$\hat{i}_t$ not close to $\tilde{i}_t$

Keep the inflation rather small!

less optimal
Interest Rate Rule:

\[ i_t = (\tilde{i}_t)^+ = \left( a_t - \gamma \sigma_t^2 (1 - |1 - \alpha_t|) \right)^+ \]

assuming optimal inflation
- Create an economic environment where

\[ a_t > \gamma \alpha_t \sigma_t^2 \implies \tilde{i}_t > 0 \]

\[ \implies \text{Avoid extended long boom with subsequent crash!} \]

No cheap credit!

otherwise economic trap with zero interest
• target inflation rate level

\[ \pi_t = a_t - c_t + \frac{\gamma \sigma_t^2}{2} \alpha_t (\alpha_t - 1) \]

after crash may be deflationary period if \( \alpha_t \in (0, 1) \)
when $\pi_t$ on target, $\tilde{i}_t > 0$, $\alpha_t < 1$

$\implies$ can steer the economy:

\[ i_t > \tilde{i}_t \Rightarrow c_t \downarrow \]

\[ i_t < \tilde{i}_t \Rightarrow c_t \uparrow \]

strongest impact on upward trend for

\[ i_t \approx \phi_t = a_t - \gamma \sigma_t^2 \]

optimal wealth evolution
when $\pi_t$ on target, $\tilde{i}_t > 0$, $\alpha_t > 1$

$\implies$ can steer the economy:

$$i_t > (\tilde{i}_t)^+ \implies c_t \downarrow$$

$$i_t < (\tilde{i}_t)^+ \implies c_t \uparrow$$

non-optimal wealth evolution

Set inflation $\pi_t$ slightly above $\tilde{\pi}_t$!

Dangerous, could become subcritical
• when credit market clears

\[ \alpha_t = 1 \]

\[ \implies \text{target inflation rate minimal} \]

least likely to have economic trap
For $c_t = c$, minimal inflation, $\alpha_t = 1 \implies$

$$U \left( \frac{cW_t}{I_t} \right) = E_t \left( U \left( \frac{cW_s}{I_s} \right) \right)$$

fair monetary policy
• benchmarked accumulated wealth

trendless

\[
\frac{W_t G_t}{S_t^*} = E_t \left( \frac{W_s G_s}{S_s^*} \right)
\]

achieved in the least expensive manner
<table>
<thead>
<tr>
<th>Country</th>
<th>$\alpha$</th>
<th>$i$</th>
<th>$\pi$</th>
<th>$\sigma$</th>
<th>$d$</th>
<th>$\theta$</th>
<th>$\gamma \alpha$</th>
<th>$c$</th>
<th>$\pi_{\text{min}}$</th>
<th>$\pi - \pi_{\text{min}}$</th>
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<td>Australia</td>
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</table>

Table 1: Estimates from sixteen markets
• market price of risk

\[ \theta \approx \frac{a - i}{\sigma} \approx 0.236 \]

• fraction times risk aversion

\[ \gamma \alpha \approx \frac{a - i}{\sigma^2} = \frac{\theta}{\sigma} \approx 1.09 \]
• consumption rate for $\alpha = 1$

\[ c \approx i - \pi + \frac{(a - i)^2}{2 \gamma \sigma^2} \approx 0.033 \approx 0.039 = d \]

• critical inflation

\[ \tilde{\pi} \approx a - c - \frac{\gamma \sigma^2}{2} \approx 0.038 \leq 0.049 \approx \pi \]
References


