Forward Hedging and Vertical Integration:
Theory and Evidence from the French Electricity Market

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Abstract

This paper analyzes the interactions between vertical integration and spot, forward and retail markets in risk management. We develop an equilibrium model for the spot, forward and retail markets, where agents are specialized in upstream or downstream segments, or both if they are integrated. Agents choose their retail market share and forward positions under uncertainty before production and supply occur.

We point out that vertical integration and forward hedging are two separate levers for demand and spot price risk diversification. We show that they exhibit similar properties relatively to their impact on retail prices and agents’ utility, but also a number of discrepancies due to the asymmetry between upstream and downstream segments. While agents always use the forward market if they are allowed to, vertical integration may not arise in equilibrium. In addition, in presence of highly risk aversion downstream agents, vertical integration may be a better way to diversify risk than spot, forward and retail markets.

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1 Introduction

Vertical integration may arise as a response to problems caused by contractual incompleteness (Grossman and Hart (1986), Bolton and Whinston (1993) and Rey and Tirole (2004)) and as ways to acquire valuable private information about the production process (Arrow (1975)), to avoid rationing (Green (1986), Bolton and Whinston (1993)) and to weaken rivals (Bolton and Whinston (1993), Rey and Tirole (2004), Chemla (2003)). Uncertainty in demand, lack of market flexibility and (implied) risk aversion may also entail vertical integration (Hendrikse and Peters (1989), Carlton (1979), Perry (1989), Emons (1996)). Specifically, vertical integration may prove efficient when the market fails to provide a full set of hedging instruments (Chao, Oren and Wilson (2005a,b)). But a number of questions remain not fully understood. To what extent and in which environments is vertical integration important? When does it take place? How can we quantify the respective effects of hedging instruments such as a forward market and the long-term risk management nature of vertical integration? The main objective of this paper is to clarify and to quantify the respective roles of forward markets and vertical integration in risk diversification and to discuss the relationship between vertical integration and forward hedging contracts.

Specifically, we aim at understanding the fundamental mechanisms of risk diversification in retail, forward and spot markets, together with the relationship linking each market’s equilibrium price. In order to focus on risk, we examine perfectly competitive markets in which agents disregard any influence they could have on prices or on the other agents’ decisions. We develop a two-date equilibrium model of perfectly competitive retail, forward and spot markets for a non-storable good. The non-storability of the good prevents the firm from benefiting from yet another possibility of “home-made risk management”, which in our model is a central feature of vertical integration. At time $t = 0$, downstream firms (or downstream subsidiaries of integrated firms) choose their retail market shares and forward positions for time $t = 1$. At that time, upstream firms (or downstream subsidiaries) produce the good, sell it to downstream entities on the spot market, in which consumers can buy. Decisions at time $t = 0$ must be taken under uncertain demand and spot price. Demand uncertainty is revealed at time $t = 1$ before production occurs. Agents have

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1Long-term contracts are in principle means of ensuring relationship-specific investments, supply and price stability. However, they are usually difficult to write so contracts end up being incomplete. Vertical integration, by granting the right to the owner of the assets to make a decision in all the cases not specified by a contract, may alleviate problems created by contractual incompleteness, such as the underinvestment problem stemming from agents’ possible opportunistic behavior (Klein (1988), Joskow (2005), and Grossman and Hart (1986)).

2See also, among others, Joskow (2005) and Rey and Tirole (2004).

3Since the good is non-storable, no production can occur at time $t = 0$ and be stored until time $t = 1$. 

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preferences defined by a mean-variance utility function, which can be thought of as a reduced form of traditional motives for risk management policy.

We derive the equilibrium prices and the quantities exchanged on the three markets in closed forms. We show that vertical integration and forward hedging are two ways of achieving risk diversification that exhibit similar properties. First, they both decrease the retail price. Second, they both enable agents with low generation capacity to corner larger market shares. Third, they both tend to decrease downstream firms’ utility when upstream firms are only partially integrated. Fourth, the impact of one of these levers on the retail price and on utilities is drastically reduced by the other.

However, discrepancies between these two levers arise because of the asymmetry between upstream and downstream firms in terms of risk-bearing: Downstream firms make decisions under uncertainty, while upstream firms respond to it. In the absence of forward hedging, vertical integration and demand elasticity, the profit of upstream firms is not affected by the retail price, whereas the downstream firms’ profit is. Therefore, downstream firms are more exposed to risk. First, vertical integration restores this symmetry while forward hedging does not. Second, vertical integration is more robust to high risk aversion in achieving risk diversification than forward hedging. Third, vertical integration can also increase downstream firms’ utility provided that they have sufficiently high risk aversion. Fourth, a non-integrated economy can be a stable equilibrium whereas a situation where no agents trade forward contracts is almost never a stable equilibrium. Finally, we show that although throughout most of the paper demand is assumed inelastic to the retail price, our main results prevail with elastic demand.

Our paper contributes to the literature on vertical integration as a risk management tool. In our setting, we take the stand to focus on competitive financial and market structure equilibria so we deliberately ignore other forms of vertical restraints or profit sharing rules. We follow in this the incomplete contracting approach in Bolton and Whinston (1993), Chemla (2003) and Rey and Tirole (2004). In our paper, we depart from the strategic interactions approach that is inherent to these papers to focus on non game-theoretic reasons for vertical integration. Our result that output prices decrease for risk management reasons different from double-marginalization (Tirole (1988)) is undoubtedly robust to a concentrated market structure, and unlike Bolton and Whinston and Rey

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4The utility function is a reduced-form of the effects of the market frictions that may prompt agents to be eager to hedge risk. See, for instance, Smith and Stulz (1984), Froot, Scharfstein and Stein (1993) and Grinblatt and Titman (2002).
and Tirole (2004), vertical integration can be welfare-improving. Such a welfare-improving effect of vertical integration in a concentrated market structure was present in Chemla (2003), where vertical integration could reduce a possible excessive degree of competition and subsequent fixed costs of setting up firms. In our paper, vertical integration is desirable because of its risk management role and through lower output prices. Our paper also provide a quantitative assessment of hedging and risk management through vertical integration. Like Chao, Oren and Wilson (2005a,b), we examine hedging and vertical integration at the same time, but unlike them we develop an equilibrium model that enables us to quantify hedging and the risk management role of vertical integration, and to highlight new determinants of vertical integration, such as the importance of risk aversion and the similarities and differences between hedging and risk management through vertical integration that we discussed above.

Our paper is also related to Allaz's (1992) and Bessembinder and Lemmon's (2002) equilibrium models of forward markets. We add a retail activity to their settings, and we consider jointly forward hedging and vertical integration as tools to manage risk. We also contribute to the recent literature on risk management through market versus non-market mechanisms. While markets enable investors to diversify easily (Doherty and Schlesinger (2002) and they are less sensitive to moral hazard (Doherty (1997)), non-market mechanisms such as reinsurance companies keep an important role. In Gibson, Habib and Ziegler (2007), the importance of non-market mechanisms stems from excessive information gathering from investors in financial markets. In our setting, the scope for vertical integration arises as complementary to hedging, and vertical integration is desirable because of the asymmetry between downstream and upstream firms and because of investors’ high levels of risk aversion.

The paper is organized as follows. We develop our model in Section 2. Section 3 solves for the equilibrium without a forward market. Section 4 examines financial market equilibria and the market structure with a forward market. We illustrate our analysis with data from the French electricity market in Section 5. Section 6 shows the robustness of our results when allowing for demand to be elastic. Section 7 concludes, and the proofs can be found in the appendix.
2 The model

2.1 The markets

We consider a set $\mathcal{P}$ of upstream firms that produce a homogeneous, non-storable good that they sell on spot and forward wholesale markets to a set $\mathcal{R}$ of downstream firms, or retailers. After sourcing on the markets, the downstream firms compete in a market for consumers whose demand $D$ is random and, for simplicity and unless otherwise specified, inelastic. Demand $D$ is described by a random variable on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The environment is perfectly competitive, in that all agents compete disregarding any influence they could have on the equilibrium price, or the other agents’ behavior. For simplicity, all agents have access to wholesale markets.

We allow for the presence of purely speculative agents (traders) who play no role in production or retail segments. We denote by $\mathcal{K}$ the set of all agents: producers, retailers and traders. Agents are not necessarily specialized in a single segment. Instead, they can own subsidiaries of different kinds. Hence the subsets $\mathcal{P}$ and $\mathcal{R}$ of $\mathcal{K}$ are possibly intersecting, leading to four different types of agents: pure retailers who buy on the markets and deliver to consumers, pure producers who produce and sell on wholesale markets, pure traders who speculate between the spot and forward markets, and integrated producers who produce, trade on the markets and also deliver to consumers.

There are two dates:

- At $t = 0$, downstream firms choose their market shares $\alpha_k \in [0, 1]$, $k \in \mathcal{R}$ and the agents take forward positions $f_k$, $k \in \mathcal{K}$ (where $f_k > 0$ represents a purchase).

- At $t = 1$, demand uncertainty is revealed. Agents take positions $G_k$, $k \in \mathcal{K}$, on the spot market (where $G_k > 0$ represents a purchase) and producers also choose their generation levels $S_k$, $k \in \mathcal{P}$. Since the good is non-storable, production can only occur at that time $t = 1$ when the demand uncertainty is observed and in which consumers buy the good.

Demand satisfaction at time $t = 1$ imposes the market-clearing constraint:

$$1 = \sum_{k \in \mathcal{R}} \alpha_k . \quad (2.1)$$

In addition, generation levels must meet demand $D$:

$$D = \sum_{k \in \mathcal{P}} S_k . \quad (2.2)$$
The spot and the forward markets are perfectly competitive so that the agents must meet the market-clearing constraints

\[ 0 = \sum_{k \in K} f_k \quad (2.3) \]
\[ 0 = \sum_{k \in K} G_k \quad (2.4) \]

Each agent aims at maximizing the sum of its payoffs on the retail, forward and spot markets:

\[ p \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - q f_k - P G_k - c_k (S_k) \mathbf{1}_{\{k \in \mathcal{P}\}}. \]

where \( p, P, \) and \( q \) denote the retail price, the spot price and the forward price, respectively, and \( c_k \) is the cost function to producer \( k \in \mathcal{P} \). The cost function is defined on \( \mathbb{R}_+ \), it is continuously differentiable, strictly convex, and it satisfies the Inada conditions \( c'_k(0+) = 0, \ c'_k(+\infty) = +\infty \).

Non-storability imposes that the net volume of good bought, sold or produced by Agent \( k \) is zero (no inventory is possible):

\[ 0 = \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - f_k - G_k - S_k \mathbf{1}_{\{k \in \mathcal{P}\}} , \]

This allows us to rewrite the profit function as:

\[ (p - P) \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} + (P - q) f_k + (P S_k - c_k (S_k)) \mathbf{1}_{\{k \in \mathcal{P}\}} . \]

The profit function is the sum of three distinct terms: The profit made by a retailer who satisfies demand \( \alpha_k D \) at retail price \( p \) by sourcing on the spot market at price \( P \); the profit made by a trader that buys \( f_k \) on the forward market at price \( q \) and that sells it on the spot market at price \( P \); and the profit made by a producer who generates a volume \( S_k \) at cost \( c_k (S_k) \) and sells it on the spot market at price \( P \).

Finally, the preferences of agent \( k \) are described by a mean-variance utility function with risk
aversion coefficient $\lambda_k$. We use the following notation for the mean variance utility function:

$$\text{MV}_{\lambda_k}[\xi] := \mathbb{E}[\xi] - \lambda_k \text{Var}[\xi].$$

Notice that $\text{MV}_{\lambda_k}$ has the inconvenience of not being monotonic, which implies possible negative prices in equilibrium. However, as shown in Markowitz (1979), it can always be seen as a second order expansion of a monotonic Von Neumann-Morgenstern utility function.

### 2.2 Equilibrium on the spot market

Producer $k$’s generation profit on the spot market can be written:

$$\text{PS}_k - c_k(S_k).$$  \hfill (2.5)

At time $t = 1$, when they enter the spot market, the agents know the realization of demand uncertainty $D$, and decisions on the retail and forward markets have already been made. The spot market competitive equilibrium is thus given by:

$$P^* = C'(D), S^*_k = (c'_k)^{-1}(P^*),$$  \hfill (2.6)

where the aggregate cost function $C$ is defined as

$$C(x) := \sum_{k \in \mathcal{P}} c_k \circ (c'_k)^{-1} \circ \left( \sum_{k \in \mathcal{P}} (c'_k)^{-1} \right)(x),$$

and satisfies $C'(x) = \left( \sum_{k \in \mathcal{P}} (c'_k)^{-1} \right)(x)$. so that the random variable $C(D) = \sum_{k \in \mathcal{P}} c_k(S^*_k(P^*))$ is the sum of the production costs over all producers.

Each producer produces at marginal cost. The equilibrium on the spot market only depends on the exogenous demand $D$ and the cost structure. It is independent of any decision taken at time $t = 0$, as a result of non-storability and demand inelasticity.

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5 This utility function can be thought of as a reduced form of a firm’s objective function that is made concave by presence of market frictions that generate motives for risk management (Smith and Stulz (1985), Froot, Scharfstein and Stein (1993), Grinblatt and Titman (2002, Chapter 20), and Tirole (2006, Chapter 5)) and that can generate regulatory constraints that may enhance regulated firms’ risk aversion.

6 This is different from Allaz (1992), where the demand elasticity to the spot price implies a dependency of the spot price to forward positions and a reduction of the market power of the producers.
The equilibrium spot price $P^*$ and the generation profit

$$\Pi_k^g := (P^* S^* - c_k (S^*_k)) 1_{\{k \in P\}} \tag{2.7}$$

act as exogenous random variables. We assume that their distribution is known to all agents. We then replace variables $P$ and $S_k$ by $P^*$ and $S^*_k$ as found previously, and we define

$$\Pi_k(p, q, \alpha_k, f_k) := \Pi_k^r(p, \alpha_k) + \Pi_k^t(q, f_k) + \Pi_k^g, \tag{2.8}$$

the profit function of agent $k$, where $\Pi_k^g$ is defined by (2.7) and

$$\Pi_k^r(p, \alpha_k) := (p - P^*) \alpha_k D 1_{\{k \in R\}}, \quad \Pi_k^t(q, f_k) := (P^* - q) f_k .$$

Here, $\Pi_k^r$ is the net retail profit derived from supplying a retail demand by sourcing on the spot market, and $\Pi_k^t$ is the net trading profit earned by buying on the forward market and selling on the spot market. Finally, $\Pi_k^g$ is the net generation profit gained by producing and selling production on the spot market.

### 2.3 Competitive Equilibrium

In order to define an equilibrium, we introduce the following two sets:

$$A := \left\{ (\alpha_k)_{k \in K} \in [0, 1]^{|K|} : \forall k \notin R, \alpha_k = 0 \text{ and } \sum_{k \in K} \alpha_k = 1 \right\}$$

$$F := \left\{ (f_k)_{k \in K} \in \mathbb{R}^{|K|} : \sum_{k \in K} f_k = 0 \right\} .$$

**Definition 2.1.** An equilibrium of the retail-forward equilibrium problem is a quadruple $(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+ \times \mathbb{R}_+ \times A \times F$ such that:

$$(\alpha^*_k, f^*_k) = \arg\max_{\alpha_k, f_k} MV_{\lambda_k} [\Pi_k(p^*, q^*, \alpha_k, f_k)], \forall k \in K .$$

This defines a simultaneous competitive equilibrium on both markets. Each agent submits a supply function specifying his levels of forward purchase and market share for each price level. Each agent chooses his supply function taking the prices as given and without taking into account
the presence of competitors. Then, the auctioneer collects all supply functions and sets the prices that ensure market clearing and demand satisfaction.

3 Equilibrium without a forward market

In this section, we derive the explicit formulation of the equilibrium without a forward market and we analyse our results. We define the profit function without a forward position as:

\[
\Pi_k^0(p, \alpha_k) := \Pi_k(p, 0, \alpha_k, 0) = \Pi_k'(p, \alpha_k) + \Pi_k''.
\]

Definition 2.1 reduces to:

**Definition 3.1.** An equilibrium of the retail equilibrium problem is a pair \((p^*, \alpha^*) \in \mathbb{R}_+ \times A\) such that:

\[
\alpha_k^* = \arg\max_{\alpha_k} MV_{\lambda_k} [\Pi_k^0(p^*, \alpha_k)], \quad \forall k \in K.
\]

3.1 Characterization of the equilibrium

Let

\[
\Pi_I^g := \sum_{k \in \mathcal{R} \cap \mathcal{P}} \Pi_k^g, \quad \Pi_r^g := \sum_{k \in \mathcal{R}} \Pi_r^g(p^*, \alpha_k^*) = (p^* - P^*)D
\]

be the aggregate generation profit realized by all integrated producers, i.e. firms running both generation and supply units, and the aggregate retail profit realized by all retailers, respectively.

Let

\[
\Lambda := \left(\sum_{k \in \mathcal{K}} \lambda_k^{-1}\right)^{-1}, \quad \Lambda_R := \left(\sum_{k \in \mathcal{R}} \lambda_k^{-1}\right)^{-1},
\]

be the aggregate risk aversion coefficients, respectively for the set of all agents and the set of all retailers. Parameter \(\lambda^{-1}_k\) can be interpreted as the risk tolerance of agent \(k\), as defined in Gollier (2004). Our equilibrium problem is similar on each market to that faced by syndicates in Wilson (1979), where an aggregate risk tolerance is defined by summing over the risk tolerances of the syndicate members, in agreement with (3.1). We only focus on interior equilibria, i.e. equilibria
where constraints $\alpha^* \in [0, 1]$ and $p^* \geq 0$ are not binding, by discarding cases where some retailers in $\mathcal{R}$ have null market shares. The equilibrium is then characterized by the following proposition.

**Proposition 3.1.** $(p^*, \alpha^*) \in \mathbb{R}^+_+ \times \text{int}(\mathcal{A})$ defines an equilibrium of the retail equilibrium problem without a forward market if, and only if,

$$
\alpha_k^* = \frac{\Lambda_R}{\lambda_k} + \frac{\Lambda_R \text{Cov}[\Pi^g, \Pi^\gamma_k]}{\text{Var}[\Pi^g]} - \frac{\text{Cov}[\Pi^g, \Pi^\gamma_k]}{\text{Var}[\Pi^g]},
$$

and $p^*$ solves the second order polynomial equation:

$$
0 = \mathbb{E}[(p^* - P^*)D] - 2\Lambda_R \text{Cov}[(p^* - P^*)D, (p^* - P^*)D + \Pi^\gamma_k].
$$

**Proof.** See Appendix □

Intuitively, firm $k$ chooses its market share in order to maximize its expected return while keeping variance as low as possible. In other words, the market shares have a home-made risk-management feature. Firm $k$ chooses a market share that increases with its risk tolerance is relatively higher than that of retailers and with the covariance between the aggregate generation profit to all integrated producers and the aggregate retail profit, but that decreases with the covariance between its own retail profit and the aggregate generation profit.

### 3.2 Equilibrium retail price

The retail price is characterized by a second order polynomial in (3.3). This equation may have several solutions or none. This is a common feature in such mean-variance settings. We argue that, if there exists two solutions, only one is relevant.

**Risk neutral case and uniqueness.** If some retailer is risk neutral, i.e. $\lambda_{k_0} = 0$ for some $k_0 \in \mathcal{R}$, the equilibrium retail price boils down to $p^0 = \frac{\mathbb{E}[P^*D]}{\mathbb{E}[D]}$. Since $P^* = C'(D)$ is a non-decreasing function of $D$, $P^*$ and $D$ are positively correlated, and the risk neutral retail price is greater than the expected spot price: $p^0 \geq \mathbb{E}[P^*]$.

**Aggregate risk aversion.** When all retailers are risk-averse, we expect the retail price to tend to the risk neutral price when the risk aversion coefficient of some retailer tends to zero. The aggregate risk aversion coefficient $\Lambda_R$ then also tends to zero. Suppose equation (3.3) has two non-negative
solutions $p_- \leq p_+$. The Taylor expansion of these roots around $\Lambda_R = 0$ can be written:

$$
p_- \simeq p^0 + \frac{2\Lambda_R}{E[D]} \left( \text{Cov}[PD, PD - \Pi_I^0] \right) - \frac{\text{Cov}^2[D, PD - \frac{1}{2} \Pi_I^0]}{\text{Var}[D]} + 2 \frac{\text{Cov}^2[D, PD - \frac{1}{2} \Pi_I^0 - p^0D]}{\text{Var}[D]} \right)

p_+ \simeq \frac{E[D]}{2\Lambda_R \text{Var}[D]}.
$$

This shows that $p_-$ tends to $p^0$ as $\Lambda_R$ tends to 0, while $p_+$ tends to infinity. Hence, $p_-$ appears as the unique relevant solution from an economic viewpoint.

**The effect of integration.** We prove that the presence of integrated producers decreases the retail price. Let $p^*_{NI}$ be the smallest solution to (3.3) with no integrated producer, and let $p^*_I$ be this solution with some integrated producers. When $\mathcal{R} \cap \mathcal{P} = \emptyset$, i.e. there are no integrated producers, equation (3.3) boils down to:

$$
0 = E[(p^*_{NI} - P^*)D] - 2\Lambda_R \text{Var}[(p^*_{NI} - P^*)D].
$$

Hence, the price is greater than the expected spot price. When some agents are integrated, i.e. $\mathcal{R} \cap \mathcal{P} \neq \emptyset$, the equation for the price is

$$
0 = E[(p^*_I - P^*)D] - 2\Lambda_R \text{Cov}[(p^*_I - P^*)D, (p^*_I - P^*)D + \Pi_I^0].
$$

Assume that one retailer, say $i$, decides to become an integrated producer. Then $\Lambda_R$ is unchanged and $\Pi_I^0 = \Pi_I^0$. Suppose also that $\Pi^r$ and $\Pi_I^0$ are negatively correlated. Then

$$
0 \geq E[(p^*_I - P^*)D] - 2\Lambda_R \text{Var}[(p^*_I - P^*)D].
$$

This condition on the correlation of $\Pi^r$ and $\Pi_I^0$ is intuitive. Since $P^* = C'(D)$, profit $\Pi_k^r$ is an increasing function of $D$. We cannot prove that the retail profit $\Pi^r = (p^* - P^*)D$ decreases $D$ but since $p^*$ is a fixed price and $P^*$ is increasing with $D$, intuitively we expect $\Pi^r$ to decrease with $D$ under normal conditions. This justifies the negativity assumption on the covariance of $\Pi^r$ and $\Pi_I^0$. The numerical application in Section 5 confirms this intuition. Inequality (3.5), combined with (3.3), shows that $p^*_I$ is lower than $p^*_{NI}$: integration tends to decrease the retail price.
3.3 Equilibrium market shares

We turn to the properties of market shares.

**Risk neutral case.** The equilibrium market shares of the risk averse retailers are given by:

\[
\alpha_k^0 = -\frac{\text{Cov}[\Pi^r, \Pi^g_k]}{\text{Var}[\Pi^r]},
\]

while the remaining demand is split among the risk neutral retailers. In particular, a risk averse retailer who has no generation unit ends up with a null market share.

Again, these market shares play a home-made risk management role. Market shares are chosen in order to ensure that the firm’s payoff minimizes risk for a given expected payoff.

**The effect of integration.** We argue that vertical integration enables a supplier to increase its market share. In the absence of integrated producers and risk-neutral suppliers, the market shares satisfy

\[
\alpha_k^* = \frac{\Lambda_R}{\lambda_k}.
\]

The market shares only depend on, and are distributed proportionally to, risk tolerances. If integrated firms enter the market, the pure retailers’ market shares become

\[
\alpha_k^* = \frac{\Lambda_R}{\lambda_k} + \frac{\Lambda_R}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi^g_I]}{\text{Var}[\Pi^r]},
\]

while the integrated firms have market shares:

\[
\alpha_k^* = \frac{\Lambda_R}{\lambda_k} + \frac{\Lambda_R}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi^g_I]}{\text{Var}[\Pi^r]} - \frac{\text{Cov}[\Pi^r, \Pi^g_k]}{\text{Var}[\Pi^r]}.
\]

Suppose once again that \(\Pi^r\) is negatively correlated to \(\Pi^g_I\). The pure retailers then see their market shares decrease while the integrated agents increase theirs. Indeed, the latter decrease their risk by increasing their investment in the retail market. In addition, although the market shares have changed in comparison to the previous case, the relative market shares among the set of pure

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\[\text{In order to satisfy the non-negativity condition of the market shares, equation 3.6 implies Cov}[\Pi^r, \Pi^g_k] \leq 0,\]

which is consistent with the assumption made in the previous subsection.
retailers remain unchanged:

\[ \frac{\alpha_i^*}{\alpha_j^*} = \frac{\lambda_j}{\lambda_i}. \]

4 Equilibria with a forward market

We now characterize the equilibrium in presence of a forward market.

4.1 Characterization of the equilibria

Let \( \Pi^e = \sum_{k \in K} \Pi_k(p^*, q^*, \alpha_k^*, f_k^*) \) be the aggregate profit to all segments, which coincides here with social welfare.

We focus on interior results. The equilibrium with a forward market is:

**Proposition 4.1.** \((p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+^* \times \mathbb{R}_+^* \times \text{int}(A) \times \mathbb{F} \) is an equilibrium of the retail-forward equilibrium problem if, and only if,

\[
\begin{align*}
\alpha_k^* &= \Lambda \frac{\text{Cov}(p^*, \Pi^e) - \text{Var}(p^*)}{\text{Var}[\Pi^e]} - \alpha_k^* \frac{\text{Cov}(p^*, \Pi^r)}{\text{Var}[p^*]} \quad (4.1) \\
\end{align*}
\]

\[
\begin{align*}
\alpha_k^* &= \Lambda \frac{\text{Var}(\Pi^e) \text{Cov}(\Pi^r, \Pi^e) - \Lambda \text{Var}(\Pi^r)}{\Delta} \text{Cov}(\Pi^r, \Pi^e) - \frac{\text{Var}[\Pi^e]}{\Delta} \text{Cov}(\Pi^e, \Pi^r - \frac{\Lambda}{\lambda_k}) \quad (4.2) \\
q^* &= \mathbb{E}[p^*] - 2\Lambda \text{Cov}(p^*, (p^* - D - C(D))] \quad (4.3)
\end{align*}
\]

and \( p^* \) is a root of the second order polynomial equation

\[
0 = \mathbb{E}[(p^* - D)] - 2\Lambda \text{Cov}((p^* - D), (p^* - D) + \Pi_f^2) \\
+ 2\Lambda \frac{\text{Cov}(p^*, (p^* - D))}{\text{Var}[p^*]} \text{Cov}[(p^* - D) + \Pi_f^2 - \frac{\Lambda}{\Lambda}(p^* - D - C(D))] \quad (4.4)
\]

where \( \Delta = \text{Var}(p^*) \text{Var}[\Pi^r] - \text{Cov}^2[p^*, \Pi^r] \).

**Proof.** See Appendix. □

We now examine the properties of equilibrium prices and positions on the retail and forward markets.
4.2 Equilibrium forward price

The equilibrium forward price satisfies:

\[ q^* = E[P^*] - 2\Lambda \text{Cov}[P^*, \Pi^e] . \]

It equals the expected spot price corrected by a risk premium term that accounts for the correlation between spot price and global profit \( \Pi^e \), and the aggregate risk aversion. This equation resembles that of other equilibrium models in a mean-variance setting, as shown in Allaz (1992). We can rewrite \( q^* \) as:

\[ q^* = E[ZP^*] \text{ with } Z := 1 - 2\Lambda(\Pi^e - E[\Pi^e]) . \]

If \( \Lambda \) is small enough to ensure that \( Z \) is always strictly positive, \( Z \) defines a change of probability and \( q^* \) is given by the expectation of \( P^* \) under a risk-neutral probability.

The equilibrium forward price only depends on retail and spot equilibrium prices. It is independent of the distribution of market shares and that of generation assets. Moreover, if some traders are risk neutral, the forward price boils down to the expected spot price \( q^0 = E[P^*] \). When the cost functions are quadratic, e.g. \( c_k(x) := \frac{1}{2}x(a_kx + b_k) \), \( a_k, b_k > 0 \), \( q^* \) can be written:

\[ q^* = E[P^*] - \frac{2\Lambda}{a} \text{Var}[P^*](p^* - E[P^*]) + \frac{\Lambda}{a} \text{Var}^{\frac{3}{2}}[P^*]\text{Skew}[P^*] , \]

where \( a^{-1} := \sum_{k \in P} a_k^{-1} \), as in Bessembinder and Lemmon (2002). The forward price increases with spot price skewness and, when the retail price is higher than the expected spot price, the forward price decreases with spot price volatility. In markets with volatile spot prices such as electricity markets, forward prices tend to be lower than the expected spot price. Nevertheless, forward prices greater than the expected spot price can occur when spot price skewness is large and positive, i.e. when large upward peaks are possible. In addition, the forward market equilibrium establishes the following relationship between retail and forward prices:

\[ p^* = \frac{E[P^*] - q^*}{2\Lambda \text{Cov}[P^*, D]} + \frac{\text{Cov}[P^*, C(D)]}{\text{Cov}[P^*, D]} . \]

In particular, higher forward prices are associated with lower retail prices and conversely.
4.3 Equilibrium retail price

The equation for $p^*$ with a forward market is similar to that without a forward market. An extra term

$$2\Lambda R \frac{\text{Cov}[P^*, (P^*-P^*)D]}{\text{Var}[P^*]} \left[ P^*, (P^*-P^*)D + \Pi_0^\theta - \frac{\Lambda}{\Lambda R} (p^*D - C(D)) \right]$$

appears with the forward market, corresponding to the hedging property of the forward market. We can still write the Taylor expansion around $\Lambda R = 0$ and show that only the smallest root $p^*_-$ of this equation is relevant, ensuring the uniqueness of the equilibrium. The retail prices also have the following properties:

- **Risk neutral price.** If some retailers are risk neutral, the retail price reduces to

  $$p^0 = \frac{\mathbb{E}[P^*D]}{\mathbb{E}[D]}$$

  as in the absence of a forward market.

- **Price in a fully integrated economy.** If all producers are integrated, i.e. $\mathcal{P} \subset \mathcal{R}$, then

  $$0 = \mathbb{E}[(p^*-P^*)D] - 2\Lambda R \text{Cov}[(p^*-P^*)D, p^*D - C(D)]$$

  $$+ 2\Lambda R \left( 1 - \frac{\Lambda}{\Lambda R} \right) \frac{\text{Cov}[P^*, (P^*-P^*)D]}{\text{Var}[P^*]} \text{Cov}[P^*, p^*D - C(D)].$$

  In the absence of pure traders, $\mathcal{R} = \mathcal{K}$ and $\Lambda = \Lambda R$, so that the equation above boils down to (3.3). Hence, the forward market has no impact on the retail price. To see this, assume that there are only $N$ integrated producers, with the same cost function and the same risk aversion coefficient. By symmetry, $S^*_k = \frac{D}{N}$, $\alpha^*_k = \frac{1}{N}$ and $f^*_k = 0$ for all $k$. The agents do not take any position on the forward market. The retail price should therefore not be affected.

This result highlights the symmetry between forward hedging and vertical integration. When the firms already diversify their risk by integrating vertically, the effect of forward hedging tends to vanish. Conversely, we will see in Subsection 5.2.3 that with a forward market, the degree of integration has little impact on the retail price.

- **The effect of forward trading.** In a partially integrated economy, $p^*_F \leq p^*_N$, i.e. the retail price with a forward market is lower than the retail price without a forward market, if,
and only if,

\[ 0 \leq \left( 1 - \frac{\Lambda}{\Lambda_R} \right) \text{Cov}^2[P^*, (p_{NF}^* - P^*)D] + \text{Cov}[P^*, (p_{NF}^* - P^*)D] \text{Cov}[P^*, \Pi^2_I - \frac{\Lambda}{\Lambda_R} \Pi^9]. \]

This condition is satisfied if no producer is integrated and if the retail income without a forward market is negatively correlated with the spot price. This is also satisfied if no retailer is integrated and \( \Lambda = 0 \) (e.g. there is a risk-neutral trader). Subsection 5.2 shows that forward hedging always reduces the retail price, but that its intensity decreases with the level of integration of the agents.

- **The effect of integration.** Let \( p_{NI}^* \) be the retail price in the absence of integration. To compare this price with the retail price with integrated firms, we substitute \( p_{NI}^* \) to \( p^* \) in the right hand side of (4.4) and study its sign. This boils down to studying the sign of:

\[
\frac{\text{Cov}[P^*, (p_{NI}^* - P^*)D]}{\text{Var}[P^*]} \text{Cov}[P^*, \Pi^2_I] - \text{Cov}[(p_{NI}^* - P^*)D, \Pi^9_I].
\]

In particular, with quadratic cost functions, we obtain:

\[
\frac{\text{Cov}[P^*, (p_{NI}^* - P^*)D]}{\text{Var}[P^*]} \text{Cov}[P^*, \Pi^2_k] - \text{Cov}[(p_{NI}^* - P^*)D, \Pi^9_k] = \frac{a^3}{2a_k} \left( \text{Var}[D^2] - \frac{\text{Cov}^2[D^2, D]}{\text{Var}[D]} \right)
\]

which is always positive. This means that the right side of (4.3) is positive for \( p_{NI}^* \) and shows that the retail price decreases in the presence of integrated firms. More generally, Subsection 5.3 shows that the presence of vertically integrated producers always reduces the retail price. This effect is nonetheless drastically reduced in comparison to when there is no forward market.

### 4.4 Positions on the forward market

In contrast with the forward price, forward positions do depend on both \( p^* \) and \( \alpha^* \):

\[
f_k^* = \frac{\Lambda}{\lambda_k} \frac{\text{Cov}[P^*, \Pi^e]}{\text{Var}[P^*]} - \frac{\alpha_k}{\lambda_k} \frac{\text{Cov}[P^*, \Pi^c]}{\text{Var}[P^*]} - \frac{\text{Cov}[P^*, \Pi^9_k]}{\text{Var}[P^*]}.
\]
The equilibrium forward position for agent \( k \) has three components. If \( k \) is a pure speculator, then the last two terms are zero. The first term can thus be interpreted as the trading component. It is the fraction \( \frac{\Lambda}{\lambda_k} \) of a constant term that involves the correlation between the global profit and the spot price. An extra supplying component is added to retailers. It is the fraction \( \alpha^*_k \) of a constant term that involves the correlation between the global retail profit and the spot price. If the retail market revenue is negatively correlated to the spot price, as we argued in the previous section, retailers take long positions on the forward market to hedge against high spot prices. Finally, an extra generation component is added to producers, involving the correlation between their generation profit and the spot price. As generation profits are positively correlated to the spot price, producers take short forward positions to hedge against low spot prices.

4.5 Positions on the retail market

The market shares satisfy:

\[
\alpha_k^* = \frac{\Lambda_R}{\lambda_k} + \frac{\text{Cov}[P^*, \Pi^r]}{\Delta} \text{Cov} \left[ P^*, \Pi^g_k - \frac{\Lambda_R}{\lambda_k} \Pi^g_I \right] \\
- \frac{\text{Var}[P^*]}{\Delta} \text{Cov} \left[ \Pi^r, \Pi^g_k - \frac{\Lambda_R}{\lambda_k} \Pi^g_I \right].
\]

When \( \Pi^g_k = \frac{\Lambda_R}{\lambda_k} \Pi^g_I \) for all \( k \in \mathcal{R} \), the market shares satisfy \( \alpha_k^* = \frac{\Lambda_R}{\lambda_k} \), as in a non-integrated economy without a forward market. This obtains, for instance, when there are no integrated firms, or when all producers are integrated and generation profits are proportional to risk tolerances. We rewrite \( \alpha_k^* \) as:

\[
\alpha_k^* = \alpha_k^0 + \frac{\Lambda_R}{\lambda_k} \left( 1 - \frac{\text{Cov}[P^*, \Pi^r]}{\Delta} \text{Cov} \left[ P^*, \Pi^g_k \right] + \frac{\text{Var}[P^*]}{\Delta} \text{Cov} \left[ \Pi^r, \Pi^g_k \right] \right),
\]

where

\[
\alpha_k^0 = \frac{\text{Cov}[P^*, \Pi^r]}{\Delta} \text{Cov} \left[ P^*, \Pi^g_k \right] - \frac{\text{Var}[P^*]}{\Delta} \text{Cov} \left[ \Pi^r, \Pi^g_k \right]
\]
is retailer \( k \)'s market share when there is a risk neutral retailer. This allows for an analysis of the deviation of market shares from the risk neutral equilibrium.

Section 5 highlights two important characteristics of market shares. First, the presence of a
forward market is a means for pure retailers to corner larger market shares. Second, the higher the level of integration of an integrated producer, the higher its market share.

4.6 Utility functions and the asymmetry between downstream and upstream segments

Without a forward market and vertical integration, retailers make market share decisions under uncertainty, while producers know the realization of demand when they make their generation decision. In addition, if the demand is inelastic to the retail price, upstream profits are independent of the retail price, while downstream revenues depend on the spot price. This asymmetry is central in our analysis.\(^8\)

As a consequence, a pure producer always benefits from trading forward contracts. Indeed, the generation profit \(\Pi^g_k\) is an exogenous random variable. Hence, when a producer bids the strategy \(\bar{f}_k(q) = 0\) for all forward prices \(q\), it receives utility \(\text{MV}_{\lambda_k}[\Pi^g_k]\), i.e. the same utility as when there is no forward market. Pure producers thus always enhance their utility with the forward market because the strategy \(\bar{f} = 0\) is admissible and yields the same utility as without one. In contrast, pure retailers have no guarantee to obtain a higher utility when forward contracts become available. Indeed, if retailer \(k\) decides to bid \(\bar{f} = 0\), it receives a utility \(\text{MV}_{\lambda_k}[\Pi^r_k]\). Nevertheless, the retail profit \(\Pi^r_k\) depends on \(p^*\), and thus on the other agents’ decisions. If the retail price \(p^*\) with forward trading differs from the retail price without forward trading, so does the agent \(k\)’s retail profit. By not taking a position on the forward market the agent may not receive the same utility as when there is no forward market. In Subsection 5.2, forward contracts decrease pure retailers’ utility. Forward contracts are thus not optional since when they are available, each retailer is individually better off contracting. All retailers trade forward contracts and they can offer lower retail prices. Nevertheless, the decrease in expected profit offsets that in variance and this risk hedging mechanism implies a decrease in utility compared to the case when there is no forward contract.

The effect of vertical integration on the agents’ utility is difficult to quantify. First, the evolution of risk aversion with the level of integration is not clear. One could argue that agents become more or less risk averse when integrating as the structure of the company is affected. Defining the risk aversion of an integrated agent knowing the risk aversions of the different subsidiaries is still a

\(^8\)An example for this is the California electricity crisis, where retailers were suffering large losses while producers were taking advantage of high spot prices.
widely open question. Wilson (1979) develops the idea that the risk tolerance of a syndicate, i.e. a group of agents, should be the sum of the agents’ risk tolerance. This idea is also suggested in our model, in the way \( \Lambda \) and \( \Lambda_R \) are defined. But one could also argue that the resulting risk aversion could be the lowest of the subsidiaries’ risk aversions. Indeed, all risky positions would be borne by the least risk averse agent. We illustrate this point in Subsection 5.3.

Like forward hedging, vertical integration decreases the agents’ utility because of its negative effect on retail prices. Nevertheless, for large risk aversions this effect can be reversed and the gain from hedging can be higher than the loss in expected profit. Finally, one aspect of vertical integration is that it breaks the asymmetry between upstream and downstream, and producers’ and retailers’ utilities are affected similarly by vertical integration.

5 Application to the electricity industry

We illustrate our analysis with historical data from the French electricity market. Electricity is a non-storable good, which fits our setting well. This market comprises a dominant former monopoly and new competitors. This is a not really in line with our perfect competition benchmark, but the fundamental mechanisms of risk diversification are robust to imperfect competition.

5.1 Methodology

We compute the retail and forward equilibria using French electricity data. Only spot prices and demand levels are publicly available. To estimate \( C \), we invert the spot price \( P^* = C’(D) \) to derive a candidate for \( C \). Demand and spot price hourly data from December 2004 to March 2005 are available on web sites www.rte-france.com and www.powernext.fr. They provide us with reliable estimates for \( D \) and \( P^* \). That winter was mild, but it was followed by a wave of intense cold in March. These recordings are showed on Figure 5.1 left. The circles are values for March. These high spot prices are indicative of the high market volatility. Nonetheless they remain highly heterogeneous since many generation units were unavailable during March’s cold wave. To deal with a time-varying cost function \( C \), we have added a constant to the demand sample, as suggested by the shape of the plots, to offset the unavailability effect.

We have regressed \( C \) on these processed samples, so that \( P^* = C’(D) \) (Figure 5.1 right). This, together with the risk aversion coefficients, enables us to compute the equilibrium.

Without loss of generality, the analysis can be performed with two agents. Indeed, equations
Figure 5.1: Demand and spot price samples (left). Processed and interpolated data (right).

(4.1) and (4.2) are linear in $\lambda_k^{-1}$ and $\Pi_k^R$ while (4.4) and (4.3) only involve the aggregate risk aversion coefficients $\Lambda$ and $\Lambda_R$.

5.2 The effect of the forward market

In this section, we compare the equilibria with and without a forward market under different environments. We consider two agents in the following scenarios:

1. Agent 1 is an integrated producer only facing competition on the retail market
2. Agent 1, unbundled, becomes a pure producer, while Agent 2 is a pure retailer
3. Agent 1 is integrated and faces the competition of a pure producer
4. Agent 1 and Agent 2 are integrated.

5.2.1 Unbundling of the retail activity

An integrated producer competes with pure retailers. From the earlier subsection, this is equivalent to competition between two agents with one integrated retailer, denoted Agent 1, and one pure retailer, Agent 2, standing for all retail competitors. This experiment examines whether the viability of retail competition with an integrated generation monopoly is viable.

In the absence of a forward market, the pure retailer is forced to source on the spot market from the integrated producer. Figure 5.2 left shows Agent 1’s market share as a function of the risk aversion coefficients of both agents. Agent 2 is limited to a very small market share, less than 2%, whatever the values of the risk aversion coefficients. Agent 2 is subject to high financial risk because it has to buy the good on the spot market and it is therefore exposed to the high spot price volatility. Agent 2 has limited possibilities to enter the retail market, all the more limited as
its risk aversion is large. There is little incentive for a non-integrated producer to enter the market.

Figure 5.2 right shows Agent 1’s market share with a forward market. Dotted mesh regions represent zones where there is no equilibrium ($q^* < 0$). In contrast to the previous case, if Agent 2 is less risk averse than Agent 1, then it can enter deeper in the retail market, up to 40%. Then the presence of the forward market allows non-integrated producers to contest the retail monopoly. The retail price remains unchanged, as shown on Figure 5.3. This is because all producers are integrated (see Section 4.3). The upstream risk is already diversified via the producers’ integrated structure.

The forward market changes the risk allocation between retailers but it does not enhance risk diversification. The equilibrium does not always exist in the forward market (dotted mesh zones in Figure 5.4). When both agents are highly risk averse, they do not agree on exchanging forward contracts. This is due to the mean-variance utility function. When the equilibrium exists, Agent 2’s
The forward position is in the range of its expected demand. More precisely, $f_2 \simeq 1.1 \alpha_2 \mathbb{E}[D]$: Agent 2 hedges its retail demand by 10% above the expected demand, whatever its risk aversion coefficient. That is, the integrated producer is better off being short even if it has to buy back part of the previously sold volumes on the spot market.

In addition, the forward price is almost always greater than the expected spot price ($E[P^*] = 37.9518$), at least for sufficiently low risk aversion. Finally, Figure 5.5 shows that the forward market increases both agents’ utilities. This figure shows the gain in the agents’ utility $\Delta U = U^F - U^{NF}$ that is due to the forward market. For convenience, we plotted the monotonic transform $\phi(\Delta U)$, where $\phi(x) := \text{sgn}(x) \log(1 + |x|)$, in order to show both the logarithm of $\Delta U$ and its sign. Both agents benefit from the forward market.

For a better understanding of the effect of forward hedging, Table 1 reports the relative gains in utility, average profit and variance (“Risk”) with $\lambda_1 = \lambda_2 = 10^{-6}$. We also computed the ratio
“excess average profit” over “excess risk”, denoted “Profit vs. Risk”, and performance measure, namely the ratio “expected profit” over risk. It appears that forward trading has a higher effect on Agent 2’s utility than on Agent 1’s. Agent 1 uses forward contracts mostly for hedging purposes, thus reducing the variance of its profits by 59 %. The relative loss in average profit associated to this hedging position is low, 0.67 times smaller than the decrease in the variance, resulting in a slight increase in utility. In contrast, Agent 2 increases both risk and its average profit by 793 %. The increase in average profit is twice the increase in risk, so its utility also increases. Forward trading has a higher effect on Agent 2’s business. Nevertheless, the ratio expected gain over risk increases more for Agent 1 than Agent 2.

<table>
<thead>
<tr>
<th></th>
<th>Agent 1</th>
<th>Agent 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility: (\frac{\Delta U}{\Delta \text{Var}})</td>
<td>8.8 (10^{-2}) %</td>
<td>793 %</td>
</tr>
<tr>
<td>Av. Profit: (\frac{\Delta \text{E}}{\Delta \text{Var}})</td>
<td>-0.17 %</td>
<td>793 %</td>
</tr>
<tr>
<td>Risk: (\frac{\Delta \text{Var}}{\Delta \text{Var}})</td>
<td>-59 %</td>
<td>793 %</td>
</tr>
<tr>
<td>Profit vs. Risk: (\frac{\Delta \text{Var}}{\Delta \text{E} \times \Delta \text{Var}})</td>
<td>0.67</td>
<td>2</td>
</tr>
<tr>
<td>Performance: (\frac{\Delta \text{Var}}{\Delta \text{E} \times \Delta \text{Var}})</td>
<td>146 %</td>
<td>(\approx 0)</td>
</tr>
</tbody>
</table>

Table 1: The impact of forward trading on utility, profit and risk, when \(\lambda_1 = \lambda_2 = 10^{-6}\).

5.2.2 Full unbundling

In this section, the generation activity is separated from the retail activity. Agent 1 is a producer while Agent 2 is a retailer (we do not consider pure traders here). In an unbundled economy, we proved that the forward market has no impact on the market shares, which only depend on the risk aversion coefficients (cf. Section 4.5). Nonetheless, the forward market affects the retail price. Figure 5.6 shows the retail price as a function of the risk aversion coefficients without (left) and with (right) the forward market (For the sake of clarity the zones with no-equilibrium are not showed). If Agent 2, the retailer, is too risk-averse, equilibrium does not exist even with the forward market. This is an illustration of the asymmetry between retailers and producers. Supply might not be sustainable in this model, whereas generation always is. Second, the forward market decreases the retail price.

Figure 5.7 shows the logarithmic transform of the gain in utility \(\phi(\Delta U)\) due to the forward market for both agents. The forward market increases the producer’s utility, while it decreases that of the retailer, unlike the integrated economy of Section 5.2.1.
As argued in Section 4, the forward strategy $f_k = 0$ is always admissible, which ensures an increase in utility for producers when forward contracts are available. The situation is different for pure retailers. While the strategy $f_k = 0$ is admissible, it is not profitable if the other retailers do trade forward contracts. In the absence of a collusive behavior where all retailers avoid trading forward contracts, the forward market does not necessarily increase their utility.

Once partially hedged on the forward market, the retailers can offer lower retail prices. In the meantime, market shares are fixed proportionally to risk tolerances, as demand is inelastic to retail price, and they are not affected by the forward market. Expanding the market shares and compensate for the loss of profit is not possible.

The gain from hedging is half the expected loss on the retail market that is induced by the decrease in the retail price: $U^F_2 - U^{NF}_2 \simeq \frac{1}{2}(p^F - p^{NF})\mathbb{E}[D]$. This explains the decrease in the retail price and in the retailers’ utility. To illustrate this argument, we computed some indicators.

Figure 5.6: Equilibrium retail price without (left) and with (right) the forward market as functions of the logarithm of risk aversion coefficients.

Figure 5.7: Gain in utility due to the forward market for Agents 1 and 2, as functions of the logarithm of risk aversion coefficients.
of risk and profit as in the previous subsection. The results are shown in Table 2. Agent 1 can now both increase its average profit by 0.88 % and decrease its risk by 99 %, hence increasing its utility by 326 %. In the meantime, Agent 2 decreases its risk, average profit and utility by 98 %, the decrease in average profit being twice the risk reduction, as above.

5.2.3 Unbundling of the generation activity

Here, generation assets are split between integrated (Agent 1) and pure (Agent 2) producers. We examine the impact of the forward market on retail prices.

In addition to the risk aversion coefficients fixed to $\lambda_1 = \lambda_2 = 10^{-6}$, we need to specify the distribution of generation assets between the agents. We choose three different methods to vary the distribution of generation assets between the two agents, each parameterized by a coefficient $0 \leq x \leq 100$. Method 1 allocates the first $x\%$ of generation capacity with the lowest marginal cost to Agent 1. Method 2 allocates the first $x\%$ of generation capacity with the highest marginal cost to Agent 1. Method 3 is halfway between the previous two: Agent 1 obtains the first $x/2\%$ of generation capacity with the lowest marginal cost and the first $x/2\%$ with the highest marginal cost. This method represents balanced competition between the two agents.

When Agent 1 has no generation capacity, we are back to the previous subsection. In this setting, the equilibrium retail price decreases with Agent 1’s total capacity (see Figure 5.8) in both cases, with and without a forward market. It is thus minimal when Agent 1 owns all the generation assets. Nonetheless, the variations of the retail price with the forward market are highly reduced. In this context, unbundling the generation monopoly increases the retail price. In addition, the forward market has a major effect on the retail price, which decreases by up to 20%.

As in paragraph 5.2.2, the forward market always increases Agent 2’s utility, while it always decreases that of Agent 1, the integrated producer (Figure 5.9). The intensity of this decrease is

<table>
<thead>
<tr>
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<th>Agent 2</th>
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<tbody>
<tr>
<td>Utility:</td>
<td>$\Delta \frac{U}{</td>
<td>U</td>
</tr>
<tr>
<td>Av. Profit:</td>
<td>$\Delta \frac{E}{</td>
<td>E</td>
</tr>
<tr>
<td>Risk:</td>
<td>$\Delta \frac{\text{Var}}{\text{Var}}$</td>
<td>-99 %</td>
</tr>
<tr>
<td>Profit vs. Risk:</td>
<td>$\Delta \frac{\text{Var}}{\text{Var}}$</td>
<td>-0.62 %</td>
</tr>
<tr>
<td>Performance:</td>
<td>$\Delta \frac{\text{Var}}{\text{Var}}$</td>
<td>117.63</td>
</tr>
</tbody>
</table>

Table 2: The impact of forward trading on utility, profit and risk, when $\lambda_1 = \lambda_2 = 10^{-6}$. 

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nonetheless reduced with integration. Pure producers prompt retailers to trade forward contracts and to decrease the retail price, thereby decreasing their utility. There is a strong asymmetry between producers and retailers.

Figure 5.9: Gain in utility for Agent 1 and Agent 2 due to the forward market, as a function of Agent 1’s proportion of total capacity.

We now fix the distribution of generation assets and let the risk aversion coefficients vary. In the absence of a forward market we obtain Figure 5.10. Agent 1 owns 70 % of generation capacity on the left, and 60 % on the right. Comparing with Figure 5.6, an equilibrium always exists if the level of integration of Agent 1, i.e. its proportion of total capacity, is high enough. This proves that vertical integration can induce better risk diversification than forward trading when retailers are highly risk averse. Vertical integration is more robust to high risk aversion.
5.2.4 Competition between integrated producers

In a fully integrated market structure, the retail price is affected neither by the forward market nor by the distribution of the generation means. As before, we start by fixing the risk aversion coefficients \( \lambda_1 = \lambda_2 = 10^{-6} \), and let the distribution of generation assets vary. Figure 5.11 shows

Figure 5.10: Equilibrium retail price without a forward market, when Agent 1 owns 70 % (left) and 60 % (right) of production capacity, as a function of risk aversion coefficients.

Figure 5.11: Agent 1’s market share without (left) and with (right) the forward market, for \( \lambda_1 = \lambda_2 = 10^{-6} \), as a function of Agent 1’s proportion of total capacity.

Agent 1’s market share as a function of its total capacity, without (left) and with the forward market. As in paragraph 5.2.1, the forward market enhances the ability of an agent that owns few generation means to take a significant position on the retail market. Both agents’ utilities increase with the total capacity, both with and without a forward market, as Figure 5.12-left shows. In this case, the agents are symmetric, and they have the same behavior. In addition, the forward market increases both agents’ utility (see Figure 5.12-right), although the increase is very small (in the range of 0.1%).
Figure 5.12: Utility without the forward market (left) and gain in utility due to the forward market (right) for Agent 1 with $\lambda_1 = \lambda_2 = 10^{-6}$.

5.3 The effect of vertical integration

We focus on the effect of vertical structure on the agents’ utility.

5.3.1 Methodology

We consider four agents, two pure retailers $R_1$, $R_2$, and two pure producers $P_1$, $P_2$. We then assume that $R_1$ and $P_1$ decide to merge, hereby creating an integrated producer $I_1$. Finally we allow $R_2$ and $P_2$ to merge, thereby creating another integrated producer $I_2$. We assume that $R_1$, $R_2$, $P_1$ and $P_2$ have the same risk aversion coefficient $\lambda$.

The relationship between the risk aversion coefficients of $I_1$, $R_1$ and $P_1$ is difficult to identify. It may depend on the synergies resulting from the merger, the cost reductions, the risk management policy in the new structure, etc. Without a forward market, the producers’ risk aversion coefficients do not affect the equilibrium. Hence, it seems reasonable to attribute a coefficient $\lambda$ to $I_1$ and $I_2$. This is no longer the case with the forward market, and the linearity of the equilibrium suggests that risk tolerances should be summed among agents of the same type. Nonetheless, when aggregating agents of different types, this argument is not clear. In the absence of a clear-cut answer, we choose to apply the same rule as in the absence of a forward market and attribute a coefficient $\lambda$ to $I_1$ and $I_2$.

In order to evaluate the benefit that agents $R_1$ and $P_1$ would have if they merge, we compare the utility $\text{MV}_\lambda[\Pi_{I_1}]$ of the resulting entity to the aggregate utility $\text{MV}_\lambda[\Pi_{R_1} + \Pi_{P_1}]$ of $R_1$ and $P_1$. In the following subsections, we refer to Agent 1 as either the pair $R_1$, $P_1$ or $I_1$ if integration took

\[9\] Results concerning retail prices and market shares are similar to those of the previous section and confirm our conclusions.
5.3.2 Vertical integration without and with a forward market

Figure 5.13 shows that when there is no forward market a firm is worse off being integrated, whatever the type of its competitors, when \( \lambda = 10^{-6} \). As with forward hedging, vertical integration induces such a decrease in the retail price that the decrease in the variance is more than offset by the decrease in the expected profit. As a result the utility decreases. Nonetheless, there is no such equilibrium effect with vertical integration that prompts the agents to vertically integrate although they suffer a decrease in utility. Here, the stable equilibrium would be that no one integrates. The great difference between vertical integration and forward trading is that vertical integration can increase utility when agents are highly risk averse.

The same computation as in Figure 5.13 with a risk aversion coefficient \( \lambda = 10^{-5.1} \) is depicted in Figure 5.14. If \( R_1 \) owns enough generation capacity, then the utility of \( I_1 \) facing \( R_2 + P_2 \) is higher than that of \( R_1 + P_1 \). Hence, there is an incentive for vertical integration when other agents are non-integrated. Similarly, Figure 5.14-right shows that the utility of \( I_2 \) facing \( I_1 \) is always higher than that of \( R_2 + P_2 \), which confers \( I_2 \) with an incentive for vertical integration in face of integrated agents. The equilibrium is then full-integration.

We now introduce a forward market and keep the risk aversion coefficients equal to \( \lambda = 10^{-6} \). Figure 5.15 shows that the effect of vertical integration is drastically reduced in an environment with a forward market. Integrating has almost no effect on utility. In addition, even for a larger risk aversion (ex: \( \lambda = 10^{-5.1} \)), the forward market reduces the incentive for vertical integration.
Figure 5.14: Utility of Agent 1 (left) and Agent 2 (right) without a forward market, as functions of Agent 1’s proportion of total capacity.

Figure 5.15: Utility of Agent 1 (left) and Agent 2 (right) with a forward market, as functions of Agent 1’s proportion of total capacity.

6 Elastic demand curve

If demand is elastic to the spot price, perfect competition on the spot market ensures that our results are unchanged because the spot market equilibrium remains independent of retail and forward equilibrium. Our model can also be solved with a demand curve that is elastic to the retail price: $D(p)$. We can then proceed as in Sections 3 or 4 and show that equations (5.2) and (4.1)-(4.2)-(4.3) remain valid. The only difference lies in the equation satisfied by $p^\star$. When the retail price is elastic, this equation becomes

\[
0 = \mathbb{E}[(p^\star - P^\star(p^\star))D(p^\star)]
- 2\Lambda_R \text{Cov}[(p^\star - P^\star(p^\star))D(p^\star), (p^\star - P^\star(p^\star))D(p^\star) + \Pi_f^2(p^\star)]
\]
in the absence of a forward market, and:

\[
0 = \mathbb{E}[(p^* - P^*(p^*))D(p^*)] - 2\Lambda_R \text{Cov}[(p^* - P^*(p^*))D(p^*), (p^* - P^*(p^*))D(p^*) + \Pi^I_{II}(p^*)] \\
+ 2\Lambda R \frac{\text{Cov}[P^*(p^*), (p^* - P^*(p^*))D(p^*)]}{\text{Var}[P^*(p^*)]} \text{Cov} [P^*(p^*), (p^* - P^*(p^*))D(p^*)] \\
- 2\Lambda \frac{\text{Cov}[P^*(p^*), (p^* - P^*(p^*))D(p^*)]}{\text{Var}[P^*(p^*)]} \text{Cov} [P^*(p^*), p^* D(p^*) - C(D(p^*))]
\]

with a forward market. This non-linear equation may be hard to solve, especially if we cannot have an explicit formulation of the spot equilibrium. Nevertheless, the equation simplifies in some cases, as we show in the following subsection.

6.0.3 Quadratic cost functions

Consider the particular case of quadratic and symmetric cost functions:

\[ c_k(x) = \frac{c}{2} x^2, \quad \forall k \in \mathcal{K}. \]

Assume that demand is a linear function of retail price of the form:

\[ D(p) = D_0 - \mu(p - p_0), \]

where \( D_0 \) is an exogenous random variable, \( p_0 \) is some non-negative reference price and \( \mu > 0 \). In this setting the equilibrium on the spot market can be solved explicitly. We obtain:

\[
\begin{align*}
S^*_k &= \frac{1}{N_p} D(p^*), \quad P^* = \frac{c}{N_p} D(p^*) \\
\Pi^I_k &= \frac{c N_I}{2 N_p^2} D^2(p^*), \quad \Pi^I = \frac{c N_I}{2 N_p^2} D^2(p^*) \\
\Pi^g &= \frac{c}{2 N_p} D^2(p^*)
\end{align*}
\]

(6.1)

where \( N_P \) is the number of producers and \( N_I \) the number of integrated producers. The retail price \( p^* \) is then given by the smallest root of a second order polynomial equation (cf. Appendix A).

6.0.4 Examples

To illustrate the effect of demand elasticity, we compute the equilibrium found above in two cases. First, we study competition between one pure retailer and one pure producer, as we did in paragraph 5.2.2. Second, we examine the case of a pure retailer and an integrated producer, as in paragraph 31.
5.2.1 To this end, we use the demand samples of the previous section. Taking expectation on both sides of the equation giving $P^*$ in (6.1), we estimate the cost function coefficient $c$ as:

$$c = N_P \frac{\mathbb{E}[P^*]}{\mathbb{E}[D]} \simeq N_P \times 5.6143 \times 10^{-4}.$$

We set $p_0 = \mathbb{E}[P] \simeq 37.95$ as the expected spot price, and we write the elasticity coefficient $\mu$ as a percentage of expected demand $\mathbb{E}[D]$.

**Pure retailer and pure producer.** Assume that Agent 2 is the pure producer and agent 2 the pure retailer. We set both risk aversion coefficients to $\lambda_1 = \lambda_2 = 10^{-6}$. Figure 6.16 right shows the equilibrium retail price with and without a forward market, as a function of $\mu$, in percentage of $\mathbb{E}[D]$. Again, the forward market decreases the retail price. Nonetheless, this effect tends to shrink as demand elasticity increases. In addition, demand elasticity decreases the retail price. As consumers respond to retail price, the retailers face low demand if they set a high retail price. They are thus prompted to decrease the retail price. Figure 6.17 shows Agent 1’s utility with and without a forward market. This utility still increases when a forward market is introduced. More interestingly, the pure producer’s utility decreases with demand elasticity. We have already mentioned the strong asymmetry between retailers and producers: producers set the equilibrium spot price and thus affect retailers’ profit, while retailers cannot affect producers’ profit. When demand is elastic to the retail price, this asymmetry is reduced and retailers affect producers’ profit via the retail price. Since the expected demand decreases with demand elasticity (see Figure 6.16 left), so does the producers’ utility. Figure 6.18 shows that Agent 2’s utility decreases with demand elasticity when there is no forward market, but that it increases with demand elasticity with
Figure 6.17: Agent 1’s utility with (right) and without (left) a forward market as a function of $\mu$.

Figure 6.18: Agent 2’s utility with (right) and without (left) a forward market as a function of $\mu$.

a forward market. When there is no forward market, the pure retailer is affected by the decrease in both demand and the retail price. With a forward contract, the pure retailer can transfer more risk to the pure producer and take advantage of demand elasticity. In particular the excess utility of Agent 2 due to the forward market increases with demand elasticity, as we suggested in Subsection 5.2.2.

**Pure retailer and integrated producer.**

In order to study the effect of demand elasticity on market shares, we consider the case where an integrated producer, Agent 1, competes with a pure retailer, Agent 2. The risk aversion coefficients of both agents are set to $\lambda_1 = \lambda_2 = 10^{-6}$. Figure 6.19-right shows Agent 1’s market share with and without a forward market. As in Subsection 5.2.1, the pure retailer cannot compete with the integrated retailer in the absence of a forward market. In this example, both agents have the same risk aversion coefficient. Agent 2 has no possibility to enter the market and he has a market share almost equal to zero. With a forward market though, Agent 2 can corner a market share of 25%. We also observe an interesting effect: market shares are not affected by demand elasticity. Although
both the retail price and expected demand decrease with demand elasticity (see Figures 6.19-left and 6.20), market shares remain unchanged. The advantage of being integrated is unaffected by demand elasticity.

**Conclusion.** Our results in earlier sections are robust to demand elasticity to retail price. In particular, our conclusions concerning the impact of vertical integration and forward trading on risk diversification remain unchanged.

## 7 Concluding Remarks

In our paper, vertical integration has a positive effect in competitive markets, especially in terms of lower retail prices. In terms of risk diversification, it exhibits properties that linear instruments such as forward contracts cannot achieve. It reduces the asymmetric risk structure between upstream and downstream. We have shown a number of mechanisms through which, as in Chao, Oren and Wilson (2005a,b), some level of integration is beneficial even in the presence of wholesale markets.
Arguably, the motive for vertical integration would be lower if the good was storable. Hence, our model predicts that other things equal, we should observe more vertical integration in markets without forward markets than in markets with forward markets, but also in markets where good are non storable than in markets with durable goods. The next step would be to examine the possibility of large agents and market power.

Allaz (1992) has pointed out that forward trading may reduce producers’ market power. In contrast, vertical integration sometimes extends market power (Bolton and Whinston (1993), Chemla (2003), Rey and Tirole (2004)). It would be interesting to study the resulting effect when these two mechanisms coexist. Future developments in the field of market equilibria should integrate those elements into the risk management analysis. Our analysis could then serve as a benchmark to quantify the effects of imperfect competition.
A Appendix

Proof of Proposition 3.1
Maximizing the mean-variance criteria over $\alpha_k$ yields the following first order condition for agent $k \in \mathcal{R}$:

$$0 = \mathbb{E}[(p^* - P^*)D] - 2\lambda_k \text{Cov}[(p^* - P^*)D, (p^* - P^*)\alpha_k^*D + \Pi_k^g],$$

which is sufficient for maximality by convexity, and gives:

$$\alpha_k^* = \frac{\mathbb{E}[(p^* - P^*)D]}{2\lambda_k \text{Var}[(p^* - P^*)D]} - \frac{\text{Cov}[(p^* - P^*)D, \Pi_k^g]}{\text{Var}[(p^* - P^*)D]}.$$

The coupling constraint $\sum_{k \in \mathcal{R}} \alpha_k^* = 1$ gives the condition on $p^*$:

$$0 = \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - P^*)D, \Pi_k^g] ,$$

and allows us to derive (3.3).

Proof of Proposition 4.1
If $k \in \mathcal{R}$, the first order condition of profit maximization reads:

$$0 = M \begin{bmatrix} f_k^* \\ \alpha_k^* \end{bmatrix} - \begin{bmatrix} \mathbb{E}[P^* - q^*] - \text{Cov}[P^*, \Pi_k^g] \\ \frac{\mathbb{E}[p^* - P^*]}{2\lambda_k} - \text{Cov}[(P^* - P^*)D, \Pi_k^g] \end{bmatrix},$$

where $M$ is the variance-covariance matrix of vector $[P^*, (p^* - P^*)D]$. By inverting the system, we obtain $f_k^*$ and $\alpha_k^*$ in terms of $p^*$ and $q^*$. If $k \notin \mathcal{R}$, $\Pi_k$ does not depend on $\alpha_k$ and the first order condition reads:

$$0 = \mathbb{E}[P^* - q^*] - 2\lambda_k \text{Cov}[P^* - q^*, (P^* - q^*)f_k^* + \Pi_k^g].$$
Market-clearing constraint (2.3) can be expressed as:

\[
0 = \sum_{k \in K} f_k^* \\
= \frac{\text{Var}[(p^* - P^*) D]}{\Delta} \left( \frac{\mathbb{E}[P^* - q^*]}{2 \Lambda R} - \text{Cov}[P^*, \sum_{k \in R} \Pi_k^2] \right) \\
- \frac{\text{Cov}[P^*, (p^* - P^*) D]}{\Delta} \left( \frac{\mathbb{E}[(p^* - P^*) D]}{2 \Lambda R} - \text{Cov}[(p^* - P^*) D, \sum_{k \in R} \Pi_k^2] \right) \\
+ \frac{\mathbb{E}[P^* - q^*]}{2 \text{Var}[P^*]} \left( \frac{1}{\Lambda} - \frac{1}{\Lambda_R} \right) - \frac{\text{Cov}[P^*, \sum_{k \in R} \Pi_k^2]}{\text{Var}[P^*]},
\]

where \( \Delta \) is the determinant of \( M \), which leads to:

\[
0 = \Delta \left( \frac{\mathbb{E}[P^* - q^*]}{2 \Lambda} - \text{Cov}[P^*, \sum_{k \in K} \Pi_k^2] \right) - \text{Cov}[P^*, (p^* - P^*) D] \text{Var}[P^*] \left( \frac{\mathbb{E}[(p^* - P^*) D]}{2 \Lambda_R} - \text{Cov}[(p^* - P^*) D, \sum_{k \in R} \Pi_k^2] \right) \\
+ \text{Cov}^2[P^*, (p^* - P^*) D] \left( \frac{\mathbb{E}[P^* - q^*]}{2 \Lambda_R} - \text{Cov}[P^*, \sum_{k \in R} \Pi_k^2] \right).
\] (A.1)

The load satisfaction constraint (2.1) can be written

\[
1 = \sum_{k \in R} \alpha_k^* \\
= - \frac{\text{Cov}[P^*, (p^* - P^*) D]}{\Delta} \left( \frac{\mathbb{E}[P^* - q^*]}{2 \Lambda R} - \text{Cov}[P^*, \sum_{k \in R} \Pi_k^2] \right) \\
+ \frac{\text{Var}[P^*]}{\Delta} \left( \frac{\mathbb{E}[(p^* - P^*) D]}{2 \Lambda R} - \text{Cov}[(p^* - P^*) D, \sum_{k \in R} \Pi_k^2] \right),
\]

which yields, using (A.1):

\[
0 = \frac{\mathbb{E}[P^* - q^*]}{2 \Lambda} - \text{Cov}[P^*, (p^* - P^*) D] + \sum_{k \in K} \Pi_k^2.
\]

As \( \sum_{k \in K} \Pi_k^2 = P^* D - C(D) \), we obtain equation (4.3). Using this result to simplify (A.1), we derive the desired (4.4) for \( p^* \). Finally, using those two equations, we can re-arrange for \( f_k^* \) and \( \alpha_k^* \) to obtain equations (4.1) and (4.2).
Equations for the retail price under elastic demand

The retail price, \( p^* \), is then the smallest root of the second order polynomial:

\[
0 = (p^*)^2 \left\{ -\mu \left( 1 + \frac{c}{N_P} \right) - 2\Lambda_R \left[ 1 + 4\mu \frac{c}{N_P} \left( 1 + \frac{c}{N_P} \right) - \mu \frac{c}{N_P} \frac{N_I}{N_P} \left( 1 + 2\mu \frac{c}{N_P} \right) \right] \text{Var}[D_0] \right\}
+ p^* \left\{ \left( 1 + 2\mu \frac{c}{N_P} \right) \mathbb{E}[D_0] + \mu p_0 \left( 1 + 2\mu \frac{c}{N_P} \right) \right\}
+ \Lambda_R \left( 2 - \frac{N_I}{N_P} \right) \text{Var}[D_0] + 2\mu p_0 \text{Var}[D_0]
\]

in the absence of a forward market, and by:

\[
0 = (p^*)^2 \left\{ -\mu - \mu^2 \frac{c}{N_P} - 2\Lambda \left( 1 + 3\mu \frac{c}{N_P} + 2\mu^2 \frac{c^2}{N_P^2} \right) \text{Var}[D_0] \right\}
+ p^* \left\{ \left( 1 + 2\mu \frac{c}{N_P} \right) \mathbb{E}[D_0] + \mu p_0 \left( 1 + 2\mu \frac{c}{N_P} \right) \right\}
+ \Lambda_R \left( 2 - \frac{N_I}{N_P} \right) \text{Var}[D_0] + 2\mu p_0 \text{Var}[D_0]
\]

with a forward market.
References


