ABSTRACT

I derive a representation of average returns implied by any conditional asset pricing model that restricts beta-dynamics to depend only on riskless rate fluctuations. These fluctuations are sufficient to measure the asymptotic effect of conditioning information on average returns because they mirror changes in the pivotal conditioning variable: the representative investor’s expected IMRS. Implementing my representation for the conditional CAPM, ICAPM, and APT, I find a remarkable improvement. For example, in striking contrast to extant evidence, the conditional CAPM explains 64% of the cross section of returns for 25 size and value portfolios and 62% for 175 anomaly-based test portfolios.

JEL classification: G12; E44

Keywords: Conditional asset pricing; CAPM; Riskless rate; Conditioning information; Size premium; Value premium; Gross profitability premium; Low beta anomaly.
1. Introduction

Asset pricing theory typically predicts a relation between expected returns and intertemporal marginal rates of substitution (IMRS) conditional on the information investors use to make investment decisions such as the current interest rate. For example, the capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965; Mossin, 1966) can be interpreted as predicting the expected returns of assets conditional on the information set of a representative investor (Jagannathan and Wang, 1996). The distinction between conditional models and unconditional estimates provided by asymptotic time series methods poses a challenge for econometricians: how to design time series tests of conditional asset pricing models that depend on an unobserved information set. For example, a challenge in testing the average returns implied by the conditional CAPM is measuring the conditional market risk premium used by investors.

Simple time series regression methods, which estimate unconditional moments, are usually insufficient for estimating average returns predicted by conditional models (Dybvig and Ross 1985; Gibbons and Ferson, 1985; Hansen and Richard, 1987; Harvey, 1989). This paper introduces a novel representation of average returns implied by asset pricing models that summarizes the information set of the representative investor using a single conditioning variable: the one-period riskless rate. The key insight that allows this parsimonious specification to be obtained for unconditional expected returns is that covariance with the investor’s expected IMRS can be measured using the covariance with the inverse of the riskless rate, which is equal to the expected IMRS under standard theoretical restrictions (see, e.g., Cochrane, 2001). Since the riskless asset is just a claim to one dollar in one-period, irrespective of the realized state of nature, its price can be used to cleanly impute covariance with the expected IMRS (see also Ferson, 1989). This feature of the riskless asset allows for the derivation of a riskless rate representation

---

1 Most asset prices that are used to impute conditioning information (such as the default spread) have different payoffs in different states of nature. Such variables are likely to be correlated with other economic variables in addition to the expected IMRS. In contrast, the riskless rate has a uniform payoff structure that gives exactly the same payoff in every state of nature, making it a powerful proxy for the conditional expected IMRS. This payoff structure also gives the riskless rate a fundamental role in a market economy: It helps balance new investments against their opportunity cost of capital, willingness to take risks across investors, and preferences to consume earlier against willingness to defer consumption (see e.g., Black, 1995).
of unconditional expected returns, the population equivalent of average returns. I show that average returns predicted by conditional asset pricing models can be estimated by simply conditioning on the historical dynamics of the riskless rate (agnostic to the source of these dynamics). Although more conditioning variables improve the estimation efficiency of unconditional expected returns, my riskless rate representation provides unbiased estimates under relatively weak assumptions. Thus, I augment the usual asset pricing relation with riskless rate covariance terms to derive an unconditional asset pricing relation and its econometric counterpart.

Implications of the conditional CAPM and conditional multifactor models are tested for observed average returns. After accounting for the unconditional covariance of asset returns with riskless rates, the conditional CAPM specification can explain 64% of the cross section of expected returns for 25 size and value portfolios and 62% for a set of 175 portfolios based on size, value, beta, momentum, variance, residual variance, profitability, and investment. When I specify an investment-based factor [conservative minus aggressive investments (CMA); Fama and French, 2015] and a hedge for future changes in interest rates (the 90-day Treasury bill return) as priced factors along with the market portfolio, this multifactor model explains 72% of the cross section of returns for 175 portfolios (with significant risk premia for the additional factors).2

One could find the improvement in explanatory power using the unconditional representation surprising. While the inverse relation between the riskless rate and the expected IMRS is well known, its fluctuations have not been considered as key determinants of the cross section of returns. A probable reason is that the magnitude of riskless rate fluctuations is considered small. It thus follows to ask: Why are large effects of these small fluctuations seen on the cross section of returns? Riskless rate fluctuations produce large cross-sectional differences in returns because these fluctuations are highly persistent.3 Although the small conditional volatility

---

2 Hou, Xue, and Zhang (2015) discuss a q-theory–based motivation for an investment-based factor such as CMA.

3 Such persistent variation in the riskless rate can arise from multiple sources including persistent consumption shocks, productivity shocks, and time-preference (asset demand) shocks. In a consumption-based model, Bansal and Yaron (2004) show that a small long-run predictable component in consumption growth can help explain key asset market phenomenon. Because the riskless rate is determined by the average consumption growth rate in such models, small long-run persistent shocks to consumption growth also reflect persistent changes in the level of the riskless rate, implying substantial unconditional variance of riskless yields. Albuquerque, Eichenbaum, Luo, and Rebelo (2015)
of riskless rate fluctuations suggests that the risk premium for these fluctuations is small, the unconditional volatility of riskless rates is orders of magnitude larger. Accordingly, I find a substantially larger improvement in explanatory power after incorporating conditioning information in the riskless rate than after incorporating hedging demands for riskless rate fluctuations.

My study makes several contributions to the literature. First, on a broader level, the paper contributes to the literature on estimating conditional asset pricing models. Prior work typically uses a scaled factor modeling approach of incorporating conditioning information in asset pricing tests (e.g., Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001). In such methods, assumptions must be made about the conditioning information that investors use. Thus, choosing an appropriate proxy for conditioning information is a challenging task as such choices are subject to concerns of data snooping (Ferson, Sarkissian, and Simin, 2008). A related issue is that scaled factor models used in such studies require a complete set of conditioning instruments. To address this issue, Lewellen and Nagel (2006) use short-window regressions to estimate the conditional CAPM. Although appropriate for estimating high-frequency beta dynamics, such an estimation strategy can be inappropriate in accounting for low-frequency variations in slow-moving betas, such as those over business cycle frequencies. My estimation strategy gets around this problem by expressing unconditional expected returns in terms of a single conditioning variable: the riskless rate, which is relatively easier to measure compared with the full set. My estimation strategy helps mitigate both data snooping and omitted conditioning variable–related issues.

Second, my paper shows that covariance of dynamic riskless rates and expected returns systematically differs across anomaly portfolios. I find that expected excess returns of portfolios with more option-like characteristics compared with assets-in-place, such as growth stocks, are more exposed to riskless rate fluctuations. This is consistent with predictions of models of the firm in which riskless rate fluctuations induce changes in the value of options-like assets [see, obtain related riskless rate dynamics in an asset pricing model with demand shocks arising from stochastic changes in agent’s time preference.]

4 Proxies for the conditional market risk premium are often motivated using business cycle variables that predict the market risk premium such as the lagged default spread (Jagannathan and Wang, 1996).
e.g., Berk, Green, and Naik (1999) and Da, Guo, and Jagannathan (2012)]. I find that accounting for heterogeneity in cross-sectional exposures to riskless rate fluctuations helps explain the average returns of various test assets.

Finally, my study helps explain the continued use of the CAPM by managers and investors to make decisions on the cost of capital conditional on the extant interest rate and beta of the firm (Graham and Harvey, 2001; Welch, 2008). Controlling for the information in the extant interest rate and its influence on firm beta (which are both known to investors and managers at the time of determining cost of capital), the CAPM produces reasonably accurate estimates of expected returns (Jagannathan and Wang, 1996; Da, Guo, and Jagannathan, 2012). Potentially resurrecting the classical CAPM, my results suggest that it describes the data remarkably well.

The paper proceeds as follows. Section 2 provides the riskless rate representation of average time-series returns. Section 3 discusses the empirical implementation of the riskless rate representation. Section 4 describes the data and presents the empirical results. Section 5 concludes.

2. A riskless rate representation of average returns

The usual objective of empirical work on asset pricing models is to explain differences in average returns. By averaging a time series of returns, consistent estimates of unconditional expected returns are obtained. These estimates are formed using no additional information about the state of the economy in which they were realized. However, asset pricing models such as the classical CAPM can also be interpreted as a description of equilibrium expected returns, conditional on information available to investors at a particular time. As average returns do not directly correspond to the predictions of conditional asset pricing models, designing tests of conditional asset pricing models without knowledge of the information set of investors is difficult. The purpose of this section is to derive a representation of unconditional expected returns (average returns) implied by any conditional asset pricing model. In Subsection 2.1, I describe the assumed economic environment. In Subsection 2.2, I derive the riskless rate representation.

2.1. Theoretical framework
Asset prices are determined by both their payoffs and their marginal rates of substitution in future states of the world. If no arbitrage opportunities arise, then a stochastic discount factor (SDF; \(M_{t+1}\)) exists such that for any asset \(i\) with return \(R_{i,t+1}\),

\[
E_t[M_{t+1}R_{i,t+1}] = 1,
\]

where \(E_t[.\] denotes the expectation conditional on information available to investors at time \(t\) (Harrison and Kreps, 1979). Eq. (1) implies that if a riskless asset exists, its yield \((R_{f,t})\) is equal to \(1/ \mu_{M,t}\), where \(\mu_{M,t} = E_t[M_{t+1}]\). This implies, other things equal, the real rate tends to fall when investors increase their precautionary savings in times of high marginal utility, and the real rate tends to rise when the IMRS is low and investors find consuming earlier more attractive.

For returns in excess of this riskless rate, Eq. (1) implies

\[
E_t[M_{t+1}(R_{i,t+1} - R_{f,t})] = 0,
\]

which provides a conditional expected return relation implied by no arbitrage:

\[
\mu_{i,t} - R_{f,t} = -R_{f,t} \text{cov}_{i}[M_{t+1}, R_{i,t+1}] = R_{f,t} \tilde{\beta}_{i,M,t},
\]

where \(\mu_{i,t}\) denotes \(E_t[R_{i,t+1}]\), \(\text{cov}_{i} [.\] denotes conditional covariance given investor’s information set (which includes \(R_{f,t}\)), and \(\tilde{\beta}_{i,M,t} = -\text{cov}_{i}[M_{t+1}, R_{i,t+1}]\) denotes the conditional covariance (beta) of any asset \(i\). The tilde in the notation is used to highlight that the beta has not been scaled by the variance of the SDF. The negative sign in the notation ensures that \(\tilde{\beta}_{i,M,t}\) is interpreted in the traditional direction (i.e., a higher beta implies a higher expected return). Replacing the conditional covariance term in Eq. (3) with the conditional covariance of the SDF with excess returns, a common practice in empirical applications, produces an identical equation for expected excess returns. The one-period riskless rate is known at the start of every period and, consequently, the conditional covariance of the SDF with returns in Eq. (3) is identical to its conditional covariance with excess returns. Using returns to derive my representation is only for notational convenience.

\[5\] The representation derived is also valid when the riskless asset does not exist. In this case, \(R_{f,t}\) represents a random variable whose value is equal to \(1/ \mu_{M,t}\).
The corresponding expected return relation in the CAPM notation is in terms of the conditional expected returns of the market portfolio \( \left( \mu_{m,t} - R_{f,t} \right) \) and uses the standard beta, which requires scaling by the conditional variance of the market portfolio \( \text{var}\left[R_{m,t+1}, R_{f,t+1}\right] \). Because neither the conditional market risk premium nor the conditional market variance is directly observable, this representation is usually applied by using short-horizon rolling windows or a proxy for the conditioning information available to investors.\(^6\) By contrast, the riskless rate representation in Eq. (3) is in terms of the riskless rate, of which the Treasury bill is usually considered a good proxy (see Appendix B). I argue that using Eq. (3) and the Treasury bill rate as proxies for the riskless rate help improve estimation of average returns implied by asset pricing models.

### 2.2. A riskless rate representation of unconditional expected excess returns

I now derive implications for unconditional expected returns allowing for dynamic riskless rates. Average returns are estimates of unconditional expected returns \( E\left[R_{t+1}\right] \). Because my aim is to explain the cross-sectional variation in average returns predicted by Eq. (3), I begin by taking unconditional expectation of both sides of the equation, accounting for time variation in the riskless rate. Assuming that the process of the riskless rate is stationary, I obtain

\[
E\left[\mu_{i,t} - R_{f,t}\right] = \bar{R}_f \beta^{i,M} + \text{cov}\left[R_{f,t}, \tilde{\beta}_{i,M,t}\right],
\]

where \( \bar{R}_f \equiv E[R_{f,t}] \) is the unconditional expected value of the riskless rate and \( \tilde{\beta}_{i,M} \equiv E[\tilde{\beta}_{i,M,t}] \) is the expected conditional beta (expected beta). When the riskless rate is time-varying, its fluctuations are likely to be correlated with the fluctuations of betas. For example, when the riskless rate is high, more leveraged firms are more likely to face financial difficulties and have higher conditional betas. The second term in Eq. (4) measures the impact of the covariance of dynamic betas and dynamic riskless rates on average returns. If the riskless rate is constant, then

---

\(^6\) For the use of short-horizon regressions, see Lewellen and Nagel (2006). For a discussion of the underlying assumptions regarding the riskless rate in standard implementations of the conditioning variable approach, see Appendix A.
the second term in the right-hand side of Eq. (4) is zero and this covariance does not influence average returns.

2.2.1. Solving for expected conditional beta \( (\overline{\beta}_{i,M}) \) using the law of total covariance

I suggest substituting the expected conditional beta term \((\overline{\beta}_{i,M})\) in Eq. (4) with quantities that are easier to estimate using ordinary least squares (OLS) regressions. For this purpose, time series regressions provide consistent estimates of unconditional beta \((\tilde{\beta}_{i,M} = -\text{cov}\left[M_{t+1}, R_{t+1}\right])\), which is equal to the slope coefficient in the regression multiplied by \(\text{var}\left[M_{t+1}\right]\). Unconditional beta estimates are formed using no information about the current state of the economy and are generally not consistent estimators of expected beta \((\overline{\beta}_{i,M} = E\left[\tilde{\beta}_{i,M, t}\right] = E\left[-\text{cov}_{t}\left[M_{t+1}, R_{t+1}\right]\right])\), which is the parameter of interest in a conditional asset pricing model. An important difference in notation between \(\overline{\beta}_{i,M}\) and \(\tilde{\beta}_{i,M}\) is that the covariance term in \(\tilde{\beta}_{i,M}\) does not have a \(t\) subscript because it represents an unconditional covariance.

The law of iterated expectations implies that to obtain an estimate of \(\overline{\beta}_{i,M}\) when the riskless rate is time-varying, an econometrician should control for \(\tilde{\beta}_{i,f} = \text{cov}\left[1/R_{f,t}, \mu_{t}\right]\), the covariance between the inverse of the riskless rate and the asset’s conditional expected return:

\[
\overline{\beta}_{i,M} = E\left[-\text{cov}_{t}\left[M_{t+1}, R_{t+1}\right]\right] = -E\left[E_{t}\left[M_{t+1}R_{t+1}\right] - E_{t}\left[M_{t+1}\right]E_{t}\left[R_{t+1}\right]\right] \\
= -\text{cov}\left[M_{t+1}, R_{t+1}\right] + \text{cov}\left[\mu_{t+1}, \mu_{t}\right] = \tilde{\beta}_{i,M} + \text{cov}\left[1/R_{f,t}\right, R_{t+1}\right] \\
= \tilde{\beta}_{i,M} + \tilde{\beta}_{i,f}.
\]

Eq. (5) says that to obtain an estimate of \(\overline{\beta}_{i,M}\), an econometrician can simply add \(\tilde{\beta}_{i,f}\) to the unconditional beta.\(^7\) The difference between expected beta \((\overline{\beta}_{i,M})\) and unconditional beta \((\tilde{\beta}_{i,M})\) is

\(^7\) The expression for \(\tilde{\beta}_{i,f}\) in terms of realized returns follows from the law of iterated expectations because realized returns are expected returns plus unexpected returns. Because unexpected returns are by definition uncorrelated with
the riskless rate beta \( \tilde{\beta}_{i,f} \), which measures the covariance of expected returns with the expected IMRS \( \text{cov} \left[ \mu_{M,t}, \mu_{t,f} \right] \). (This relation between expected covariance and unconditional covariance is sometimes referred to as the law of total covariance.)

2.2.2. The riskless rate representation of expected average excess returns

From Eqs. (4), and (5), I obtain a general representation of unconditional expected returns:

\[
E \left[ \mu_{i,t} - R_{f,t} \right] = \bar{R}_f \tilde{\beta}_{i,M} + \text{cov} \left[ R_{f,t}, \tilde{\beta}_{i,M,t} \right] + \bar{R}_f \tilde{\beta}_{i,f}. \tag{6}
\]

Comparing the unconditional relation given by Eq. (6) with the conditional relation given by Eq. (3) highlights two key differences. First, the \( \text{cov} \left[ R_{f,t}, \tilde{\beta}_{i,M,t} \right] \) term enters the relation because it summarizes the conditioning information used by the representative agent that influences average returns. Eq. (6) also implies that if the real riskless rate is observable, accounting for the covariance of \( \tilde{\beta}_{i,M,t} \) with the dynamic riskless rate is sufficient to specify unconditional expected returns because any variable that predicts the expected IMRS must also predict the inverse of the observed riskless rate. Second, when the riskless rate is dynamic, the covariance between conditional expected returns and riskless rate enters the unconditional expected return relation. Its contribution to average returns is \( \bar{R}_f \tilde{\beta}_{i,f} \).

How large can the contribution of these additional terms be, when the observed variation in the riskless rate is small? The magnitude of influence of the additional terms depends on the persistence of riskless rate fluctuations. I assume that the riskless rate process is stationary but highly persistent (this assumption is tested in Section 3). If this process were a near-unit root process, the variance of the riskless rate and, consequently, the influence of the additional terms related to the covariance with the riskless rate can be substantial. Small persistent shocks to the

---

the known riskless rate, the covariance of inverse of the riskless rate with realized returns simply estimates \( \tilde{\beta}_{i,f} \).

Also, here the tilde notation of \( \tilde{\beta}_{i,f} \) represents covariance with \( 1/R_{f,t} \), not covariance with \( -R_{f,t} \).
SDF are consistent with a substantial unconditional variance of the riskless rate (see also Bansal and Yaron, 2004).

2.2.3. Restricted beta dynamics implied by the riskless rate representation

Re-writing the asset pricing Euler equation as Eq. (6) helps focus on the relevant components of beta dynamics that determine average returns. To see this, note that the term $\text{cov} \left[ R_{f,t}, \tilde{\beta}_{i,M,t} \right]$ in Eq. (6) depends only on the component of $\tilde{\beta}_{i,M,t}$ in the linear span of the dynamic riskless rate. This observation can be used to decompose $\tilde{\beta}_{i,M,t}$ into orthogonal components by projecting the conditional beta on relevant components of riskless rate dynamics. To illustrate, consider the decomposition:

$$\tilde{\beta}_{i,M,t} = \tilde{h}_{i,t} + b_{i,t} R_{f,t} + e_{i,t},$$

where $e_{i,t}$ denotes the change in beta that is orthogonal to $R_{f,t}$. The riskless rate representation implies that $e_{i,t}$, the time-dynamics of firm $i$ orthogonal to riskless rate fluctuations, can be safely ignored when specifying the average return relation for the firm. The component of $\tilde{\beta}_{i,M,t}$ that influences average returns is the one that co-varies with the riskless rate.

Therefore, an important implication of Eq. (6) is that econometricians do not need to specify the dynamics of time-varying betas using arbitrary conditioning variables and can use a linear projection on the dynamic riskless rate instead. While this insight dramatically reduces the set of arbitrary variables available to specify beta dynamics, it still leaves open a subset of choices that requires further economic judgement regarding why a firm’s beta covaries with the riskless rate. Next, I discuss a parsimonious representation of beta dynamics that allows for both high- and low-frequency covariance of asset returns with the riskless rate.

It follows to ask, what are the likely sources of beta covariation with the riskless rate? In Appendix C, I discuss the predictions of investment-based asset pricing theory (e.g., Berk, Green, and Naik, 1999) for the covariance of conditional expected returns with the riskless rate. The basic insight is that there are two fundamental components of riskless rate dynamics that have

---

8 This is analogous to the decomposition in Jagannathan and Wang (1996) where they note that their unconditional asset pricing relation depends only on the part of conditional beta that is in the linear span of the market risk premium. Based on this observation, they specify a conditional CAPM specification using a projection of the conditional beta on the market risk premium.
heterogeneous influence on asset returns: the persistent level component \((R_{f,t-1})\) and the recent changes component \((\Delta_{f,t})\). The value of firms with more assets in place tend to covary more with the level of the riskless rate in the previous period \((R_{f,t-1})\) and the value of firms with more option-like characteristics tend to covary more with recent changes in the riskless rate \((\Delta_{f,t} = R_{f,t} - R_{f,t-1})\).

To see this, note that when the level of the riskless rate \((R_{f,t-1})\) is higher than its unconditional mean (all else equal), it depresses the present value of cash flows from assets-in-place to be below their long-term mean. Therefore, if deviations of the riskless rate from its long-term mean are persistent, then the deviation of net present value of these future cash flows from their long-term means are likely to be persistent as well. Further, these present values from assets-in-place are expected to revert to their long-term means when the riskless rate approaches its long-term mean. This implies, all else equal, firms with higher assets-in-place (those with a stationary stream of cash flows) are likely to have higher future expected returns when interest rates are above their long-term means (e.g., early 1980s) and lower future expected returns when interest rates are below their long-term means (e.g., 2009–2014). In contrast, small firms, which have more value arising from growth options than assets-in-place, should be more sensitive to recent changes in the present value of the investment amount (the strike price of these growth options). This in turn is sensitive to recent changes in the riskless rate \((\Delta_{f,t})\).

I argue that recognizing the difference in covariance of firm value with the lagged and recent changes component of the riskless rate helps improve estimates of unconditional covariance and, consequently, improves cross-sectional tests. To account for the heterogeneity in beta exposure to the riskless rate, I suggest a decomposition of \(\tilde{\beta}_{i,M,t}\) that depends on the lagged riskless rate \((R_{f,t-1})\), the “recent” one-period change in the riskless rate \((\Delta_{f,t})\), and an orthogonal component \(\eta_{i,t}\) that captures the residual influence of other conditioning variables:

\[
\tilde{\beta}_{i,M,t} = \tilde{\beta}_{i,0} + \tilde{\beta}_{i,\Delta} \Delta_{f,t} + \tilde{\beta}_{i,R} R_{f,t-1} + \eta_{i,t},
\] (7)
where $\Delta_{f,t}$ represents the recent change in the riskless rate; $\bar{b}_{i,0}$ is an intercept term; $\bar{b}_{i,\Delta}$ and $\bar{b}_{i,j}$ measure the sensitivity of beta with $\Delta_{f,t}$ and $R_{f,t-1}$; and $\eta_{i,t}$ represents residual changes in beta that are on average zero and uncorrelated with $\Delta_{f,t}$ and $R_{f,t-1}$.\(^9\) I report empirical evidence in support for this decomposition in Section 4.

In summary, the riskless rate representation given by Eqns. (6) and (7) implies two key restrictions in estimating conditional asset pricing models: (i) the covariance terms measured using $\bar{b}_{i,j}$ and $\bar{b}_{i,\Delta}$ are sufficient to measure the influence of beta dynamics on average returns and, (ii) the orthogonal component $\eta_{i,t}$ does not influence the asymptotic average return of an asset.

### 3. An empirical model incorporating dynamic riskless rates

In an economic environment in which the riskless rate is dynamic, average excess returns depend not only on the unconditional covariance of an asset with the SDF, but also with the unconditional covariance of conditional beta with dynamic riskless rates and the unconditional covariance of conditional expected returns (that depend on beta and risk premia) with dynamic riskless rates. In this section, I specify an empirical model based on the riskless rate representation that can be used to implement any linear conditional asset pricing model.

Consider a single-factor representation of a linear IMRS: $M_{t+1} = a_{v,t} - b_{v} R_{v,t+1}$, where $R_{v,t+1}$ is the return on the single-factor that is perfectly correlated with shocks to the IMRS.\(^10\) The riskless rate representation given by Eq. (6) and (7) provides an asset pricing relation for expected average excess returns for this IMRS:

$$E\left[R_{i,t+1} - R_{f,t}\right] = \lambda_{i} \tilde{\beta}_{i,v} + \lambda_{v,\Delta} \tilde{\beta}_{i,v,\Delta} + \lambda_{v,\Delta} \tilde{\beta}_{i,v,\Delta} + \lambda_{v,\Delta} \tilde{\beta}_{i,v,\Delta} + \lambda_{v,\Delta} \tilde{\beta}_{i,v,\Delta}$$

\(^9\) If the persistent parameter is known, then the process could be decomposed into the expected and unexpected surprises to the riskless rate. However, because this parameter is not known and because the predictions of conditional beta of this decomposition are identical to the predictions of Eq. (7), I avoid introducing additional error-in-variables–related issues and instead use the specification given by Eq. (7).

\(^10\) Huberman and Kandel (1987) show that every multifactor factor model has a single factor representation. For notational convenience, I use the return on this single factor ($R_v$) to represent the return on the candidate multifactor model being implemented to estimate betas.
where $\lambda$ represent unconditional premium parameters and $\tilde{\beta}_{i,v}, \tilde{\beta}_{i,\Delta}$ and $\tilde{\beta}_{i,l}$ in Eq. (8) represent scaled versions of the covariance terms, $\tilde{\beta}_{i,t}, \tilde{\beta}_{i,\Delta}$ and $\tilde{\beta}_{i,l}$, with $R_{v,t-1}$ instead of with $M_{v,t-1}$ in Eqns. (6) and (7).

3.1. Estimating betas for the riskless rate representation

“Multivariate betas” ($\beta$) obtained from the following regression equation provide an equivalent representation of Eq. (8):

$$R_{it-1} - R_{f,t} = \beta_{i,0} + \beta_{i,v} (R_{v,t-1} - R_{f,t}) + \beta_{i,\Delta} (R_{v,t-1} - R_{f,t}) \Delta_{f,t} + \beta_{i,l} (R_{v,t-1} - R_{f,t}) R_{f,t}$$

$$+ \beta_{i,f} (R_{f,t} - \bar{R}_f) + \beta_{i,\Delta} (\Delta_{f,t} - \bar{\Delta}_f) + \epsilon_{i,t-1},$$

where $\bar{R}_f$ represents the sample average of the riskless rate, and $\bar{\Delta}_f$ represents the sample average of changes in the riskless rate. The last beta term ($\Delta_{f,t}$) in Eq. (9) helps absorb the potential bias in the interaction terms.\(^\text{11}\) As a consequence of its inclusion in the regression, it measures the influence of the high-frequency component of $\tilde{\beta}_{i,f}$. The $\beta_{i,f}$ term consequently captures the residual low-frequency component of $\tilde{\beta}_{i,f}$.

The corresponding riskless rate representation estimated using multivariate betas ($\beta$) estimated using Eq. (9) is:

$$E[R_{it-1} - R_{f,t}] = \lambda_v \beta_{i,v} + \lambda_\Delta \beta_{i,\Delta} + \lambda_l \beta_{i,l} + \lambda_f \beta_{i,f} + \lambda_\Delta \beta_{i,\Delta},$$

A benefit of using this representation [in comparison to Eq. (8)] is that multivariate betas correspond to estimates from a standard OLS regression. To keep the estimation parsimonious, I follow the standard practice of using multivariate regressions to estimate betas.\(^\text{12}\) I also note that the correlation between $\Delta_{f,t}$, $R_{f,t}$, and other variables in the regression influence the signs of $\lambda_f$ and $\lambda_\Delta$.

\(^{11}\) Ferson, Sarkissian, and Simin (2008) find that it is better to specify time-varying intercept terms in regression models with interaction terms, especially when persistent lagged variables are interacted with returns. The $\Delta_{f,t}$ term in Eq. (9) is sufficient for this purpose, as, along with $R_{f,t}$, it absorbs the potential bias in both interaction terms.

\(^{12}\) Jagannathan and Wang (1996) show that using univariate or multivariate beta to estimate Eq. (8) provides identical forecasts of expected returns. A disadvantage of this choice is that my unconditional risk premium estimates do not correspond to variance of the corresponding regression variable in Eq. (7) multiplied by the riskless rate.
3.2. **Proxy for the one-period riskless rate**

The conditional expected value of the stochastic discount factor depends on the nature of information available to investors and how they make use of it. The riskless rate, if it is observed by an econometrician, can be used to proxy for conditioning variables as it summarizes the information available to investors about the conditional expected value of their SDF. Using the riskless rate representation circumvents the need to choose a conditioning instrument vector (often empirically motivated) for time variation in expected returns in scaled factor models. Eq. (6) provides a modelling strategy that circumvents the requirement of specifying such a conditioning vector and proposes use of the dynamic riskless rate instead. This representation relation is valid for any stochastic discount factor and is independent of preference assumptions. For example, it is equally valid for the CAPM and the intertemporal CAPM (ICAPM).

To implement the riskless rate representation of unconditional expected returns, I need to model the relation between the expected returns of an SDF and an observable riskless rate or a valid proxy (instrument) for it. A widely used proxy for the one-period riskless rate in empirical asset pricing studies is the short-term Treasury bill yield. While this asset is practically riskless in nominal terms, it is nevertheless subject to some inflation risk. That is, changes in the short-term Treasury bill yield can be attributed to movement in the short-term real interest rate, in expected inflation, or in the inflation risk premium. Using the short-term Treasury bill yield as a proxy for the riskless rate in Eq. (6) is prudent because the inflation risk premium is generally regarded as more of a concern for long-term bonds than for the short-term interest rate (Mishkin, 1990).

Further assumptions under which the covariance of nominal excess returns with the nominal Treasury bill rate is identical to the covariance of real excess returns with the one-period real riskless rate are discussed in Appendix B. The main assumption required is that the Fisher hypothesis holds: The effects of inflation are symmetric across all my test assets. This is a reasonable and standard assumption for the analysis of equity models. To keep the empirical analysis tractable, I assume that it holds.
4. Empirical tests incorporating dynamic riskless rates

Using my empirical model for average returns, I now test the magnitude of influence of a dynamic riskless rate on unconditional estimates obtained from various asset pricing models.

4.1. Econometric method

Whether data are consistent with the asset pricing models can be determined in several ways. A benefit of using the cross-sectional test approach (e.g., Fama and MacBeth, 1973) to the time series approach (e.g., Gibbons, Ross, and Shanken, 1989) is that the cross-sectional approach allows for tests of asset pricing models that depend on traded factors as well as non-traded factors. Because the sources of unconditional covariance in the riskless rate representation are not necessarily traded, I use the two-stage cross-sectional regression (CSR) method proposed by Fama and MacBeth (1973).13

As is common in the literature, I use OLS $R^2$ in the cross-sectional regression as an informal measure to compare the relative performance of different empirical specifications. Roll and Ross (1994) show that this cross-sectional relation is very sensitive to the choice of factors, which can be close to each other in the mean-variance space and yet still produce significantly different cross-sectional slopes (positive, negative, or zero). This suggests that, even when the influence of the riskless rate on unconditional expected returns is small, ignoring its small (but systematic) impact can have a large impact on the ability of the model to explain the cross section of returns. In a related study, Levy and Roll (2010) find that small variations of the sample parameters, well within their estimation error bounds, can make a typical market proxy efficient. As the inclusion of the additional betas implied by my model is motivated by theoretical restrictions that help make the unconditional relation more precise, I believe it is prudent to include them in cross-sectional tests of any conditional asset pricing model.

13 The generalized method of moments (GMM) can also be used for cross-sectional tests. Ferson and Foerster (1994) point out that GMM coefficient estimates and standard errors can be highly unreliable in complex conditional asset pricing models, such as those with time-varying betas. Because the Fama and MacBeth (1973) procedure does not depend on unknown weights that are being estimated, it is likely to improve the statistical properties of such tests in finite samples. This is also an additional benefit of testing the unconditional implications of these conditional models (see, e.g., Jagannathan and Wang, 1996).
I am also interested in examining whether various coefficients of the models are different from zero after accounting for estimation error. For this purpose, I calculate standard errors using the approach suggested by Shanken (1992).

4.2. Data

So far, the notation “R” has been used to denote gross returns. Because the data are measured in percentages, I use the notation \( r_t = (R_t - 1) \times 100 \) for percentage returns.

4.2.1. The short-term Treasury bill rate

Data for yields and returns used in my study are obtained from Kenneth R. French’s website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). The one-month yields \( f_{t,1} \) are based on the average of bid and ask prices for Treasury bills. Table 1, Panel A, shows the means, standard deviations, and first six autocorrelations of the one-month yield, the monthly change in the one-month yield, and the monthly return on the 90-day Treasury bill, downloaded from the Center for Research in Security Prices. The data cover the period from July 1963 to December 2014.

[Insert Table 1 near here]

The average level of the one-month yield is 0.41%, with an estimated standard deviation of 0.26%. The innovations to the Treasury bill yield are highly persistent. It’s first order autocorrelation is 0.97. This persistence suggests the presence of econometric issues in estimating the standard deviation of the riskless rate. The sample variance estimates of such a persistent random variable is downward-biased as the time series observations are not independent (e.g., Newey and West, 1987). Assuming an AR(1) process, the variance of the population variance of the Treasury bill yield is given by \( \text{var}[r_{f,1}] = \frac{1}{(1 - \rho^2)} \text{var}[e_{f,1}] \), where \( e_{f,1} \) denotes surprises to its yield. If \( \rho \) is near one, as in the case of the Treasury bill yield, then the true variance can be significantly larger than the estimated sample variance in the data. If the true value of \( \rho \) is one, then the process of the Treasury bill yield is not stationary and its unconditional variance is unbounded. I reject a unit root for the one-month Treasury bill yield using the Phillips-Perron
This is consistent with asset pricing models that typically model the short-rate process as stationary.

Fig. 1 plots the Treasury bill yield and changes to this yield. The persistence in the yield is evident from the figure. Changes to the yield \( \Delta_{f,t} = r_{f,t} - r_{f,t-1} \) are much less persistent. The assumption that the riskless rate fluctuations are strongly persistent is not invalidated by the presence of volatility clustering in \( \Delta_{f,t} \). I estimate an autoregressive integrated moving average (ARIMA) model with time-varying volatility of the residuals in Table 1, Panel B, and find that the change in the autoregressive (AR) coefficient after allowing the innovation to be heteroskedastic is small.

Table 1, Panel C, shows the correlations and associated \( p \)-values of changes in the Treasury bill rate with its lagged value and measures of changes in the inflation rate. I use two proxies for inflation (\( \pi \)): the percentage change in the US Consumer Price Index (CPI) and the percentage change in the Core CPI (all items in CPI less food and energy). Monthly data on inflation are from the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of St. Louis.

The correlations between changes in the inflation rate and changes in the riskless rate are small. Only a small proportion of the changes in the Treasury bill yield can be explained by change in inflation. In addition to theoretical considerations, this weak empirical relation suggests that estimates of covariance of asset excess returns with changes in the Treasury bill yield are unlikely to be driven by the expected inflation component of the Treasury bill yield.

4.2.2. Test portfolios

Market returns, factor returns, and test portfolio returns downloaded from French’s website are constructed based on market equity, book-to-market, market beta, momentum (past

---

14 The choice of the Phillips-Perron test is based on its ability to address the issues related to heteroskedasticity and unspecified autocorrelation in the disturbance process that likely influences the augmented Dickey-Fuller test for the riskless rate (Phillips and Perron, 1988).

15 To check for consistency with the evidence in Fama and Schwert (1977, 1979), I perform a subperiod analysis for the period before and after January 1980. Although the relation between changes in core CPI inflation and changes in the Treasury yield is significantly stronger before than after 1980, the \( R^2 \) is still lower than 3%.
performance), variance, residual variance (idiosyncratic risk), profitability, and investments. Depending on the context, I use either five portfolios constructed using a univariate sort on a characteristic, 25 portfolios constructed using double-minors on market equity and one other characteristic, or eight long-short factor portfolios. Details of how these portfolios are constructed can be found on French’s website.

4.3. Covariance of average equity returns with fluctuations in riskless rates

I begin my empirical analysis by examining the relation between the changes in the riskless rate and the expected excess return of the aggregate equity portfolio (the market portfolio):

$$E_t\left[r_{m,t+1} - r_{f,t}\right] = \theta_0 + \theta_{\Delta} \Delta_{f,t} + \theta_\eta \eta_{m,t},$$

(11)

where $\theta_0$ is a constant, $\theta_{\Delta}$ and $\theta_\eta$ are projection parameters, and $\eta_{m,t}$ is a component of the conditional risk premium that is orthogonal to the riskless rate. These parameters are estimated by replacing market expected excess returns at time $t$ in Eq. (11) with the realized excess returns at time $t+1$. [For a discussion of this methodology and its finite sample properties, see Ferson, Sarkissian, and Simin (2008).] Also, for ease of interpretation, I approximate the inverse of the riskless rate as $1/R_{f,t} \approx 1 - r_{f,t}$. The approximation error is expected to have small implications for average returns because the magnitude of the second order terms is typically small. The results are robust to performing the regressions using the inverse of the riskless rate.

In Table 2, Panel A, I estimate $\theta_\eta \approx -1.17$ and $\theta_{\Delta} \approx -8.52$. That is, a 25 basis point increase in the Treasury bill yield predicts a $\theta_{\Delta} \times 0.25 \approx 2.13\%$ decrease in the expected market return for the next month. Also, times when the level of the riskless rate is about 1% higher than its unconditional mean are associated with about $\theta_\eta \approx 1.17\%$ lower expected excess market returns compared with its unconditional mean. These results are stable when controls for realized inflation ($\pi_t$), the contemporaneous change in the riskless rate ($\Delta_{f,t+1}$), or the contemporaneous return on 90-day Treasury bill ($r_{90,t+1}$) are added.
Bernanke and Kuttner (2005) find related evidence of the influence of changes in interest rates on market returns. They show that a hypothetical unanticipated 25 basis point cut in the federal funds rate target is associated with a 1% increase in broad stock indexes and that the largest part of this response of stock prices to unanticipated monetary policy actions is through its effect on expected real excess returns. This is likely a conservative estimate of the association between real short rates and real excess returns due to the monetary component of Fed announcements.

My estimate of a stronger association (about 2% versus 1%) is consistent with changes in real interest rates representing changes in the expected value of the IMRS, of which the expected market portfolio return is a fundamental determinant.

Fig. 2 plots the predicted expected market risk premium using parameters estimated from Table 2, Panel A. Substantial variation exists in the conditional market risk premium. The low riskless rate in the sample post-2008 reflects the near-zero nominal interest rate in this period. The market risk premium reached the highest level after the 2008 crisis and has stayed there ever since. By contrast, low risk premia are evident during the early 1980s recession. In my framework, these two recessions have very different implications for average returns. Such implications of time variation in risk premia for the cross section of average returns are captured by my empirical model.

4.4. Cross-sectional differences in exposure to riskless rate fluctuations

My empirical model allows firms to have heterogeneous exposure to the riskless rate process. For my riskless rate representation to help improve the description of the cross section of average returns, there should be significant cross-sectional differences in exposures to the riskless rate across firms. In the Berk, Green, and Naik (1999) framework, firm characteristics such as size, book-to-market, and past returns are correlated with their exposures to interest rates and time variation in beta due to optimal investment decisions. For example, small firms, which have more

\[ \text{Insert Fig. 2 near here}\]

16 Bernanke and Kuttner find that monetary policy actions have a real effect on the market risk premium, even though stocks are claims on real assets. Even though monetary policy affects the short-term real interest rate, it has only a transitory effect on future real interest rates. Bernanke and Kuttner find that it effects future expected excess returns of stocks, which also implies a correlation between real short rates and future real expected returns.
value arising from growth options than assets-in-place, should be more sensitive to changes in the present value of the investment amount (the strike price of these growth options).

I analyze cross-sectional differences in exposure to various components of the riskless rate using eight long-short portfolios constructed based on characteristics known to produce large spreads in average returns. Do characteristics that predict higher abnormal returns (given a valid conditional SDF) also predict higher covariance with various components of the riskless rate?

For this purpose, I calculate the time series parameters of Eq. (9) for the following long-short portfolios: high market beta minus low market beta (BETA), small minus big (SMB), value minus growth (HML), winners minus losers (WML), high volatility minus low volatility (VOL), high residual variance minus low residual variance (IVOL), robust minus weak profitability (RMW), and conservative minus aggressive investments (CMA).

I use the Fama and French (2015) five-factor portfolios as proxies for SMB, HML, RMW, and CMA and the Carhart (1997) factor portfolio for WML. The remaining three portfolios (BETA, VOL, and IVOL) are constructed using the return on the highest quintile minus the lowest quintile portfolios (ranked based on univariate sorts) downloaded from French’s website.

My baseline tests examine the unconditional implications of the conditional CAPM. This SDF is assumed to have the functional form

\[ M_{t+1} = \frac{1}{R_{f,t}} - \gamma \left( R_{m,t+1} - E_t \left[ R_{m,t+1} \right] \right), \]  

where \( R_{m,t+1} \) is the return on the aggregate wealth (market) portfolio. For this SDF, Eq. (3) can be written in terms of the conditional variance or conditional expected returns on the market portfolio (Cochrane, 2001):

\[ \mu_{t,t} - R_{f,t} = \gamma \beta_{i,m,t} R_{f,t} \text{var}_t \left[ R_{m,t+1} \right] = \beta_{i,m,t} \left( \mu_{m,t} - R_{f,t} \right), \]  

where \( \beta_{i,m,t} = \text{cov}_t \left[ R_{i,t+1}, R_{m,t+1} \right] \text{var}_t \left[ R_{m,t+1} \right] \). Even under the null that the conditional CAPM holds, Eq. (6) predicts that assets with nonzero covariance with dynamic riskless rates will appear to have nonzero alphas in time series regressions. The well-known nonzero alphas of my test portfolios from the perspective of the static CAPM are replicated in Table 2, Panel B. The long and short legs of these test portfolios have significant differences in exposure to the riskless rate. To test whether the magnitude of exposure of assets to dynamic riskless rates is sufficient to
explain the abnormal returns on these portfolios, estimates are needed of the $\lambda$ parameters in Eq. (8). I estimate these parameters later using cross-sectional regressions for 175 test portfolios and find that $\hat{\lambda}_t = 0.27$ and $\hat{\lambda}_\Delta = -0.04$ (see Table 4, Panel A). Here I assume these estimates are consistent and use them to calculate the implied alpha $\hat{\alpha}_i = \hat{\beta}_{i,0} - \hat{\beta}_{i,\Delta} \hat{\lambda}_\Delta - \hat{\beta}_{i,\Delta} \hat{\lambda}_\Delta$ for various long-short portfolios. Because these parameters are estimated, the $t$-statistics reported in Table 2, Panel B, are conservative estimates.\(^{17}\)

4.4.1. High beta minus low beta

In one of the earliest tests of the CAPM, Black, Jensen, and Scholes (1972) find that time series alphas decline with beta and that the lowest beta stocks have significantly positive alphas (the low beta anomaly). More recently, Baker, Bradley, and Wurgler (2011), Baker, Bradley, and Taliaferro (2014), and Frazzini and Pedersen (2014) find evidence that this phenomenon continues in the US and is also observed internationally, across industries, Treasury markets, corporate bonds, and futures markets.\(^{18}\) Table 2, Panel B, reproduces the low beta anomaly using the static CAPM. I find that the abnormal returns ($\alpha$) associated with the high beta minus low beta anomaly are negative and significant. I also find significant differences in $\hat{\beta}_{i,\Delta}$ between the high beta and low beta portfolio. This suggests that unconditional covariance between the riskless rate and conditional expected returns could explain the low beta anomaly.

The low beta anomaly’s alpha from the static CAPM can be explained by noting that, when interest rates are rising ($\Delta_{f,} > 0$, e.g., in the late 1970s), market expected returns are decreasing. When the interest rates are falling (e.g., post-2008), market expected returns are increasing. This implies, all else equal, a positive $\hat{\beta}_{i,\Delta}$: High beta firms should have a larger decrease in conditional expected returns than low beta firms when interest rates are rising and a larger increase in conditional expected returns when interest rates are falling. Under the null of

---

\(^{17}\) The true estimation error in alpha is larger than that obtained when the true values are replaced by estimated value in the regression. In other words, when the true value of these parameters are not known, ascertaining whether alpha is significantly different from zero is more difficult.

\(^{18}\) Baker, Bradley, and Wurgler (2011) claim that institutional investors’ mandates to beat a fixed benchmark discourage arbitrage activity, which can partly explain the observed patterns in returns. Frazzini and Pedersen (2014) explain this using a model with leverage and margin constraints.
the conditional CAPM, a positive $\hat{\beta}_{i,t}$ of the BETA portfolio (and a zero $\hat{\beta}_{i,t}$) implies that the portfolio has lower average returns than those predicted by the static CAPM, which could explain the negative alpha for this portfolio. Further, when the alpha of this portfolio is adjusted using the riskless rate representation, the abnormal returns on the high beta minus low beta portfolio are not significantly different from zero. [Using short-window estimates and instrumental variables methods, Cederburg, and O’Doherty (2015) find results consistent with mine.]

4.4.2. Small minus big

In the sample, the static CAPM alpha for SMB is not significantly different from zero. This remains true when I estimate the alpha using the unconditional CAPM.

4.4.3. Value minus growth, robust minus weak profitability

Value stocks (those with high book-to-market ratios) have higher average returns than predicted by the static CAPM (Fama and French, 1992). Novy-Marx (2013) finds that more profitable firms generate significantly higher returns than unprofitable firms, despite having significantly higher valuation ratios. Further, he finds that controlling for profitability dramatically increases the performance of value strategies. Consistent with these results, Table 2, Panel B, shows significantly positive alphas using the static CAPM for both the HML and the robust minus weak profitability (RMW) portfolios. The alpha for RMW is almost three times that for HML.

The average returns on these spread portfolios based on value and profitability can be explained from the perspective of the unconditional CAPM. The returns on HML and RMW have significantly positive $\hat{\beta}_{i,t}$. When the riskless rate is above its unconditional mean (e.g., 1980), the expected returns and betas of these value and robust profitability portfolios are below their long-term means. If the current values of these firms are lower than their unconditional means, then these values are expected to rise in the future, implying a higher long-run expected return for these portfolios when interest rates are higher (e.g., the 1980s). This positive $\hat{\beta}_{i,t}$ of expected returns with the riskless rate suggests that value firms and more profitable firms have higher estimated average returns (not higher abnormal returns) due to the covariance of these firms’
conditional expected returns with the riskless rate. Also, consistent with exercise of real options by growth firms and weak profitability firms, the negative $\hat{\beta}_{i,ml}$ of these portfolios implies that when riskless rates are higher than their long-term mean, the future betas of $HML$ and $RMW$ portfolios are likely to decrease. After accounting for these unconditional betas, smaller (insignificant) alphas are estimated for both $HML$ and $RMW$.

4.4.4. Volatility and residual volatility sorted-portfolios

In an influential paper, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatility (with respect to the Fama and French three factors) have abnormally low returns. $VOL$ and $IVOL$, long-short portfolios constructed based on total volatility and idiosyncratic volatility, respectively, have significant negative $\hat{\beta}_{i,ml}$. This suggests that, when interest rates are low, stocks with higher volatility or residual volatility are likely to have higher future betas (e.g., post-2008). This is consistent with the prediction of the Berk, Green, and Naik (1999) model that options are exercised in times of low interest rates because the opportunity cost of capital is low. Because firms with higher volatility and residual volatility are likely to have higher real option components of firm value, they are also more likely to invest in new projects when the opportunity cost of capital is low. After making these investments, the risk of these firms is likely to increase to the level of assets-in-place, predicting a rise in future beta. Accounting for the beta dynamics of these portfolios, the alphas implied by the unconditional CAPM are smaller and insignificantly different from zero.

4.4.5. Winners minus losers

In the Berk, Green, and Naik (1999) framework, lagged expected returns predict future expected returns because the composition and systematic risk of the firm’s assets are persistent, which leads to momentum effects over the medium term. In my representation, the effect of these persistent expected returns is measured by positive covariance with the riskless rate. Although evidence exists of significant positive $\hat{\beta}_{i,ml}$ and $\hat{\beta}_{i,m6}$ of momentum portfolio returns with the riskless rate, the magnitude of this covariance is not sufficient to explain the large abnormal returns on the momentum spread portfolio (see Table 2, Panel B). This suggests that my specification of the unconditional CAPM is unlikely to explain the level of return spread between
winners and losers, consistent with studies such as Lewellen and Nagel (2006). The relatively frequent rebalancing required to construct the WML portfolio could induce covariance with the riskless rate at different frequencies (such as the past 12 months, which is the horizon used to sort stocks into winners or losers). However, I avoid searching alternative frequencies of covariation with the riskless rate and leave a more detailed analysis of this possibility to future research.

4.4.6. Conservative minus aggressive investments

Another challenging portfolio for the unconditional CAPM is CMA, a spread portfolio constructed based on past investments (see Fama and French, 2015). I obtain a large alpha on CMA using both the static CAPM and the unconditional CAPM. Hou, Xue, and Zhang (2015) construct a related factor based on $q$-theory. They argue that firms investing less are those firms with higher costs of capital and that firms investing more are those firms with lower costs of capital. The large and significant unconditional alpha on the CMA portfolio suggests that these differences in costs of capital across conservatively and aggressively investing firms are not driven by heterogeneous conditional exposures to the aggregate market portfolio or the riskless rate. This is consistent with the hypothesis that, in addition to the market portfolio, an investments-related priced factor exists in the economy. If an additional priced factor related to investments exists, an additional portfolio that hedges these shocks would be required to explain the returns on the CMA portfolio and on firms that are exposed to such shocks.\footnote{A possible theoretical foundation for such a common factor in returns is related to investment-specific technology shocks (IST; Kogan and Papanikolaou, 2014). In this framework, firms that experience positive IST shocks increase investments and those that receive negative IST shocks decrease them. In the Kogan and Papanikolaou model, a positive investment-specific shock reduces the cost of new capital goods, thereby increasing the net present value of future projects and the value of growth opportunities for firms.} The possibility of an additional priced investment-based factor is discussed in Subsection 4.6.

4.5. Cross-sectional tests of the conditional CAPM

I now test the riskless rate representation of Eq. (13). I first report cross-sectional test results for each of seven sets of 25 portfolios constructed based on quintiles of market equity and quintiles of book-to-market, market beta, momentum (past performance), variance, residual variance (idiosyncratic risk), profitability, and investments. I choose such a large set of test portfolios to address concerns raised by Lewellen, Nagel, and Shanken (2010), who suggest that
tests of asset pricing models can be improved by expanding the set of test portfolios beyond the 25 size and book-to-market portfolios.

The first row in each panel of Table 3 represents a test of the static CAPM. I estimate a vector of betas ($\beta_i$) using time series OLS regressions of $r_{i,t+1}$ on the market portfolio ($r_{m,t+1}$). The second row in each panel reports the isolated contribution of components of the riskless rate process in explaining the cross section of expected returns. In the third row of each panel, I estimate the full model.

In each panel of Table 3, once the unconditional implications of the conditional CAPM using the riskless rate representation are specified, the explanatory power of the conditional CAPM dramatically increases in comparison with the static CAPM. Much of this explanatory power comes from capturing the time variation of beta related to interest rate dynamics. For the 25 size and book-to-market portfolios, the explanatory power of the CAPM increases to 64%, compared with 3% for the static CAPM.

Fig. 3 plots the average returns obtained in the sample of four sets of test portfolios against their conditional CAPM implied unconditional expected returns. Consistent with the increase in explanatory power shown in Table 3, the unconditional CAPM fits the data better than the static CAPM. Also, consistent with the alpha analysis in Table 2, Panel B, these figures identify the most difficult to price portfolios as the smallest portfolios with the highest investment rates and the smallest portfolios with the most extreme past returns.

Table 4 reports the estimates from a single cross-sectional regression with the complete set of 175 test portfolios. The results are similar. The unconditional representation of the classical CAPM explains 62% of the cross section of average returns, substantially larger than the static CAPM, which explains only 5%.

Overall, the results illustrate how modeling the implications of small variations in the riskless rate can dramatically change inferences regarding the ability of a conditional asset pricing model to explain the cross section of average returns. Consistent with this increase in explanatory
power, Fig. 4 shows that the unconditional CAPM fits the data remarkably better than the static CAPM.

4.5.1. Alternative conditional CAPM models

In Table 4, Panel B, I compare the performance of my conditional CAPM specification to alternate specifications by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001). When the default spread is used as a conditioning variable, I find a $R^2$ of 12% in explaining the cross section of average returns for the 175 test assets. When the Jagannathan and Wang (1996) conditional CAPM specification with the default spread as an conditioning variable for the market risk premium and per-capital labour income growth as proxy for the returns on human capital (HC) is used, this $R^2$ increases to 23%. Using the Lettau and Ludvigson (2001) conditioning variable $CAY$ (an estimate of the unobservable consumption-aggregate wealth ratio) produces an $R^2$ of 36% for my test assets. Adding labour income growth to the model increases the $R^2$ to 40%. The fit of my conditional CAPM specification is substantially higher than these benchmark conditional models, suggesting that the riskless rate in my specification better summarizes conditioning information than these alternate specifications.

4.5.2. Structural break and subsample analysis

Empirical studies of the term structure suggest that a shift in Federal Reserve monetary policy in 1979 resulted in a structural break in the interest rate process (see, e.g., Clardia and Friedman, 1984). Table 4, Panel C, considers whether the improvement in specification for the CAPM using the riskless rate representation of unconditional expected returns is driven by a particular subsample (pre- or post-1980). This proves not to be the case, and the parameter

---

20 The construction of per-capita labor income growth series follows Jaganathan and Wang (1996). The data on personal income and population are taken from the National Income and Product Account of the U.S.A. published by the Bureau of Economic Analysis.

21 Quarterly data for $CAY$ are downloaded from http://faculty.haas.berkeley.edu/lettau/data_cay.html. To keep the regressions comparable to monthly frequency regressions, I use the quarterly value of $CAY$ known at the start of every quarter for each month in that quarter. In unreported tests, I compare the results in Lettau and Ludvigson (2001) with results using my monthly version of their model. I find that both specifications have an $R^2$ in the range of 70-75% for the 25 size and BTM portfolios, suggesting that the choice of frequency is not of first-order importance in determining the explanatory power of their model. In unreported tests, I also find a significant relation between the conditional beta series of long-short portfolios predicted using $CAY$ and that using the riskless rate.
estimates and $R^2$ are stable across the two subsamples. The results thus are not driven by any one of these two subsamples.

4.5.3. Controls for inflation

Fama and Schwert (1977) test the Fisher hypothesis under the assumption of a constant real riskless rate. They find that expected nominal return on government bonds and bills vary one-to-one with the expected inflation rate. However, they also find a negative relation between expected inflation for common stock returns (see also Schwert, 1981). Fama (1981), Kaul (1987), and Boudoukh, Richardson, and Whitelaw (1994) argue that inflation is a proxy for expectation of real variables such as output and find that, when both inflation and a measure of future real output are included as explanatory variables, this negative relation disappears. The proxy explanation is consistent with the evidence presented in this paper that a time-varying real interest rate has a negative relation with market risk premia, which is also correlated with future real output.

To test whether inflation has additional conditioning information, beyond its correlation with the real rate, I use it as a conditioning variable in Table 4, Panel D. The signs and coefficients of my unconditional CAPM specification are stable, and inflation is not a significant determinant of the cross section of average returns. This suggests that the additional information in inflation is not driving the results and that the correlation of the Treasury bill yield with the real riskless rate is the important driver of the results.

4.6. Cross-sectional tests of multivariate models

To illustrate the application of the riskless rate representation to multifactor models, I now analyze a natural multivariate extension of the CAPM specification. Because the known riskless rate at the start of every period is an important conditioning variable, it follows to consider whether unexpected changes to the riskless rates are priced by long-term investors in a multifactor model.

4.6.1. ICAPM with the riskless rate as a state variable

---

22 I do not calculate the real rate as the Treasury bill yield minus the realized inflation. Since realized inflation is measured with error, subtracting it from the Treasury bill yield is likely to introduce an additional error-in-variables bias in the regression.
My CAPM specification is compared with a static ICAPM specification and a conditional ICAPM specification. Consider an SDF that contains an additional priced factor compared with the CAPM, that is, the unexpected changes to the riskless rate:

\[ m_{t+1} = 1 - r_{f,t} - \gamma^x \left( r_{m,t+1} - E_t \left[ r_{m,t+1} \right] \right) - \gamma^h \left( r_{h,t+1} - E_t \left[ r_{h,t+1} \right] \right), \]

(13)

where \( r_{h,t+1} \) is a hedge portfolio that is perfectly correlated with a state variable describing the investment opportunity set. This SDF is motivated by the formula in Merton (1973) that expresses excess returns on any asset as a linear function of its covariance with the market and with an asset \( r_{h,t+1} \) that is perfectly correlated with changes in interest rates (see also Maio and Santa-Clara, 2015).

In the static ICAPM specification, I add the unexpected changes in interest rates as a state variable. The risk premia and betas are assumed to be constant in this specification. I test this parsimonious version of the ICAPM in which the returns to the 90-day Treasury bill \( \left( r_{T_{90},t+1} \right) \) is a hedge portfolio for shocks to the investment opportunity set. The excess return on this portfolio reflects unexpected changes to the future one-period riskless rate.

In Table 5, Panel A, I estimate a static ICAPM relation with the future change in the riskless rate as a state variable. Irrespective of whether the return on the 90-day Treasury-bill is included in the regression, or the actual change, the explanatory power of the static ICAPM is small.

Next, I estimate a conditional ICAPM relation using the riskless rate representation. The results indicate that investors do seem to ask for a significant risk premium for hedging against future changes in interest rates. However, the improvement in the explanatory power after accounting for this additional premium is small. This small magnitude is consistent with the small conditional variance of short-term interest rates, which drives conditional risk premia and a relatively large unconditional variance of riskless rates.

The key result here is that accounting for demands to hedge future changes in interest rates in the theoretical model does not drive the results. Although the riskless rate representation of unconditional expected returns in Eq. (6) resembles multi-beta asset pricing models, this
resemblance is misleading. Eq. (6) is not a special case of a multi-beta conditional asset pricing model. Dynamic models such as the Merton (1973) ICAPM allow for a time-varying interest rate and solve for the asset demands of investors who want to hedge against future changes in interest rates and other variables that describe the investment opportunity set. Because of the hedging demands arising in a dynamic economy, the conditional expected return on an asset typically is jointly linear in the conditional market beta and hedging portfolio betas. However, ICAPM investors would not want to hedge against a quantity they already know: the one-period riskless rate. Investors could want to hedge against the risk of unexpected changes in interest rates at $t+1$, but not the known level of the riskless rate at time $t$.

4.6.2. Comparison with the Fama and French factors

I now compare my model with two benchmark empirical factors models: the widely used Fama and French (1993) three-factor model and its recent extension, the Fama and French (2015) five-factor model. My model has a higher $R^2$ than the three-factor model and has a comparable $R^2$ to the five-factor model, which explains 60% of the cross section of average returns. The unconditional representation of the classical CAPM thus is at least as good as this empirically motivated five-factor model, which is in stark contrast to explanatory power of the static CAPM. From the perspective of those who believe that the market beta should influence average returns, the results highlight the benefit of using the riskless rate representation to test the conditional CAPM.

4.6.3. Unconditional and conditional APT

Motivated by the substantial explanatory power of the Fama and French (2015) five-factor model and the large and significant alphas estimated for $CMA$ in Subsection 4.4, I now add $CMA$ to the market portfolio. When only the $CMA$ portfolio is added to the market portfolio, the ability of the static arbitrage pricing theory (APT; Ross, 1976) model to explain the cross section of returns is low (5%). However, when I estimate the unconditional implications of the APT, I find a substantial improvement in the specification (70%). This unconditional test helps identify the presence of additional priced factors related to investments in a simple two-factor conditional APT setting.
Finally, I add the 90-day Treasury bill return to the specification and find that both these factors are priced and produce the highest adjusted $R^2$ (72%) of all specifications considered. This three-factor conditional APT wins the horse race for adjusted $R^2$ amongst the models considered in this paper.

5. Conclusion

Asset managers, traders, and analysts alike seek conditioning information, closely dissecting economic indicators and policy decisions, to update their beliefs regarding interest rates, market risk premia, and asset expected returns. I show that the asymptotic effects of such conditioning information on asset prices can be measured using only the information in the riskless rate process. When the investment and consumption opportunity set observed by investors changes, they adjust their portfolio choice and consumption streams. In this process, the one-period riskless rate reveals their expected IMRS. Using this feature of the riskless rate, I provide a representation for average returns implied by any conditional stochastic discount factor.

I examine the unconditional implications of the classic CAPM, ICAPM, and APT using my riskless rate representation and find substantial evidence in favor of using this representation to estimate conditional models. In fact, the results suggest that the low explanatory power of the CAPM in standard tests could largely be due to these tests ignoring the effect of riskless rate fluctuations on average returns. Nevertheless, I find that, in addition to the market portfolio, investors demand a premium to hold assets exposed to changes in future interest rates and to an investments-related common factor in stock returns. Further, these effects are not easily identified if the information in the riskless rate process is ignored. This evidence suggests that the riskless rate representation of unconditional expected returns derived in this paper helps better describe the dynamics of asset prices.

My work leaves open questions. Most notably, my representation describes average returns but is silent on the fundamental determinants of riskless rates fluctuations and the mechanism through which these fluctuations influence average returns. I leave these and other interesting questions for future research.
References


31


Appendix A. Riskless rate assumptions in the traditional conditioning approach

In this Appendix, I review the scaled representation of a stochastic discount factor used to formulate tests of asset pricing models (e.g., Hansen and Singleton, 1982; Cochrane, 1996) with the aim of highlighting underlying assumptions regarding the riskless rate.

Hansen and Singleton (1982) describe a method to estimate equations such as Eq. (2) using a generalized instrumental variables procedure. The dependence of parameters on conditioning information observed by investors is modeled by scaling the factors using a vector of instruments \( z_t \), observed by econometricians. They note that, from the standpoint of implementing this method, a researcher is given considerable latitude in selecting \( z_t \).

This framework is often applied by assuming the SDF is linear (\( M_{t+1} = a_t + b_t f_{t+1} \)), where \( b \) and \( f \) are \( K \times 1 \) vectors. It incorporates conditioning information in asset pricing tests by scaling the factors \( f_{t+1} \) by a \( J \times 1 \) vector of instruments \( z_t \) for the information set of agents at time \( t \) (including a constant). If these conditioning variables satisfy \( E[f_t] = E[z_t] = E[f_{t+1}|z_t] = 0 \), then Eqs. (1) and (2) can be expressed using a scaled SDF \( M_{t+1}^* \) as

\[
M_{t+1}^* = 1/R_{f,t} + b_z'(f_{t+1} \otimes z_t),
\]

where \( b_z \) is a constant \( KJ \times 1 \) vector and \( \otimes \) represents the Kronecker product.\(^{23}\)

Applying the scaled SDF in Eq. (14) provides an unconditional relation that can be used to specify average returns:

\[
E\left[ M_{t+1}^* R_{f,t+1} - M_{t+1}^* R_{f,t} \right] = E\left[ \frac{1}{R_{f,t}} (R_{f,t+1} - R_{f,t}) \right] + E\left[ b_z'(f_{t+1} \otimes z_t) (R_{f,t+1} - R_{f,t}) \right] = 0.
\]

\(^{23}\) The Kronecker product \( f_{t+1} \otimes z_t \) multiples every element in vector \( f_{t+1} \) with every element in vector \( z_t \). See Cochrane (2001) for a discussion about such specifications. He also examines why the assumption of linearity in conditioning information is not necessarily restrictive in this setting.
Even though evidence exists in real financial markets that the Treasury bill, a proxy for the riskless rate, significantly changes over the sample period analyzed in empirical asset pricing studies, the effect of these changes is typically considered small. If these fluctuations in riskless rates can be ignored, then the first term in Eq. (15), which measures the cross-moment of excess returns with the inverse of the riskless rate, is negligible. Using this approximation, the usual scaled factor model specification of asset pricing tests ignores the first term.

I argue that even small changes in this fundamental price can have large systematic effects on asset pricing tests and inferences because the riskless rate reflects the state of the investment and consumption opportunity set observed by investors. Consequently, I derive an expression for unconditional expected returns taking into account the first term in Eq. (15).
Appendix B. Estimating covariance with the riskless rate

When a riskless asset exists, the riskless rate is given by $R_{f,t} = 1/\mu_{M,t}$, where $\mu_{M,t} = E_t[M_{t+1}]$. The presence of the riskless rate in various expressions in section 1 is solely due to this relation. In principle, I can substitute $R_{f,t}$ in Eq. (6) with any variable that proxies for $1/\mu_{M,t}$. For example, I can use the no-arbitrage relation to derive an alternative expression for a Treasury bill yield ($R_{b,t+1}$) subject to inflation risk in terms of a hypothetical asset with riskless real yield equal to $R_{f,t}$:

$$R_{f,t}^* = E_t\left[R_{b,t+1}\right] = \frac{1}{E_t[M_{t+1}]} - \frac{\text{cov} \left[M_{t+1}, R_{b,t+1}\right]}{E_t[M_{t+1}]} = R_{f,t} + \lambda_{\pi,t},$$

where $\lambda_{\pi,t}$ is an inflation risk premium to hold the Treasury bill, which has no nominal risk but is exposed to inflation risk. $R_{f,t}^*$ is the real yield on the nominal Treasury bill. As per Eq. (16), the nominal short-term Treasury yield no longer provides an exact proxy for real riskless rate. This raises the question: If the Treasury bill is being used to proxy for the riskless rate in Eq. (6), is the time variation in inflation, the time variation in the inflation risk premium, or the time variation in the riskless rate, or some combination of the three, being accounted for? The answer depends on how the time variation in inflation and the inflation risk premium are correlated with excess returns. The Fisher hypothesis suggests that this correlation should be zero, which further implies that the Treasury bill rate can be used to calculate the covariance of an asset’s expected excess return with the expected value of the SDF.

Fisher (1930) notes that the nominal interest rate ($\tilde{\mu}_{f,t}$) can be expressed as the sum of an expected real return and an expected inflation rate. Fama and Schwert (1977) and Chowdhry,
Roll, and Xia (2005) apply this argument to all assets and in all time periods. Following this interpretation of the Fisher hypothesis, I model nominal conditional expected returns of all assets as

\[
\tilde{\mu}_{j,t} = \mu_{j,t} + \pi_t + \lambda_{\pi,t}.
\]  

(17)

This says that the market used all possible information available to correctly assess expected pure price inflation \(\pi_t\) in the next period and its associated inflation risk premium \(\lambda_{\pi,t}\). The market also determines the appropriate equilibrium expected real return \(\mu_{j,t}\) on all assets. The market then sets the price of all assets so that their expected nominal returns \(\tilde{\mu}_{j,t}\) is the sum of the equilibrium real return, the correctly assessed expected inflation rate, and the associated inflation risk premium [Eq. (17)].

To mitigate the influence of time variation in expected inflation under these assumptions, I can estimate covariances with expected excess nominal returns. If the adjustment to change nominal payoffs to real payoffs is the same for every asset, then the expected excess nominal return over the Treasury bill on any asset is identical to its expected real excess return: \(\mu_{j,t} - R_{f,t} = \tilde{\mu}_{j,t} - \tilde{R}_{f,t}\). That is, the expected excess returns on assets do not carry a component related to pure price inflation or an inflation risk premia and, consequently, the unconditional covariance of these excess returns with the Treasury bill rate is equal to that with the real interest rate. That is,

\[
\text{cov}[\mu_{j,t} - R_{f,t}, R_{f,t}] = \text{cov}[\tilde{\mu}_{j,t} - \tilde{R}_{f,t}, \tilde{R}_{f,t}].
\]  

(18)

Models in which inflation realizations could asymmetrically influence real asset returns, such as term structure models, require further structure to disentangle inflation risk
from the real interest rate (see, e.g., Ang, Bekaert, and Wei, 2008). Under such assumptions, the application of Eq. (6) would require further assumptions. Tests of unconditional expected returns given by Eq. (6) using the Treasury bill yield as a proxy for the riskless rate are more likely to be consistent with conditional asset pricing models formulated to explain equity risk premia with unpriced (or negligible) inflation risks. To keep the empirical analysis tractable, I assume that the effects of inflation are symmetric across all my test assets.
Appendix C. Investment-based asset pricing models and the dynamic riskless rate

To describe the intuition underlying the reduced-form model of beta dynamics, I turn to investment-based models of firms that decompose the value of a firm into the value of assets-in-place and the value of real and financial options.\(^{24}\)

When firms optimally exercise a real option by investing in new projects, then the beta of the firm moves nearer to the beta of assets-in-place. If the firm invests in a higher (lower) beta project, then the post-investment beta of the firm is higher (lower). If an increase in interest rates predicts an increase (decrease) in investment due to the exercise of real options, then they also predict an increase (decrease) in future beta of such firms.

Now I discuss the sensitivity of real options to the riskless rate before they are exercised. Although I do not assume that the Black and Scholes (1973) assumptions hold, for illustrative purposes I examine the covariance between dynamic interest rates and option value implied by the model. The model implies that an option’s interest rate sensitivity (\(\rho\)) is given by

\[
\rho(R_f, K, T, \sigma, S) = KTe^{-R_fT} N(-d_2),
\]

where

\[
d_2 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{S}{K} \right) + \left( R_f - \frac{\sigma^2}{2} \right) T \right].
\]

Eq. (19) says that the instantaneous sensitivity of the option-like component of firm value to the interest rates is positive. When riskless rates increase, the value of the option-like component of firms is likely to increase. Also, the sensitivity of the option-like component in firm value interacts nonlinearly with changes in interest rates, and this interaction depends on

\(^{24}\) Berk, Green, and Naik, (1999) decompose expected returns depending on interest rate sensitivity of cash flows from assets-in-place (proxied by the book-to-market ratio) and a component due to growth options, whose value depends on the interest rate level.
firm-level characteristics. The covariance of firm value with changes in interest rate is likely to be heterogeneous and depend on the macroeconomic variables that determine the current level and change in the riskless rate as well as on firm characteristics such as leverage, scale of investments, asset volatility, and life of the options available to firms.
Table 1
Summary statistics and model fit for the short-term interest rate

The variable \( r_{f,t} \) denotes the yield on the Treasury bills maturing in one month; \( r_{f,t} - r_{f,t-1} \), the associated monthly yield change; and \( \rho_j \), the estimated autocorrelation coefficient of order \( j \). Means, standard deviations, and autocorrelations of monthly Treasury bill yields, yield changes, and the 90-day T-bill monthly returns \( (r_{T90,t}) \) are computed in Panel A. Panel B presents estimates from autoregressive integrated moving average (ARIMA) models. \( a_0 \) and \( a_1 \) denote autoregressive conditional heteroskedasticity (ARCH) parameters. Panel C presents correlations and \( p \)-values of changes in the monthly yield, lagged changes in monthly yield, changes in inflation \( [\pi, \text{US Consumer Price Index (CPI)}] \), and changes in core CPI \( (\pi^*, \text{all items excluding food and oil}) \). The sample period is from July 1963 to December 2014.

Panel A: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
<th>( \rho_5 )</th>
<th>( \rho_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{f,t} )</td>
<td>0.41</td>
<td>0.26</td>
<td>0.97</td>
<td>0.95</td>
<td>0.93</td>
<td>0.92</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>( r_{f,t} - r_{f,t-1} )</td>
<td>-0.00</td>
<td>0.06</td>
<td>-0.16</td>
<td>-0.11</td>
<td>0.05</td>
<td>-0.10</td>
<td>-0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>( r_{T90,t} - r_{f,t-1} )</td>
<td>0.04</td>
<td>0.09</td>
<td>0.28</td>
<td>0.04</td>
<td>0.04</td>
<td>0.11</td>
<td>-0.01</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: ARIMA models of the Treasury bill yield

<table>
<thead>
<tr>
<th>Model</th>
<th>Intercept</th>
<th>( r_{f,t-1} )</th>
<th>( \sigma_e )</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.011</td>
<td>0.971</td>
<td>0.004</td>
<td>(1.681)</td>
<td>(110.573)</td>
</tr>
<tr>
<td>AR(1) — ARCH(1)</td>
<td>0.013</td>
<td>0.960</td>
<td>0.002</td>
<td>0.751</td>
<td>(3.205)</td>
</tr>
</tbody>
</table>

Panel C: Correlation matrix

<table>
<thead>
<tr>
<th>Variable</th>
<th>( r_{f,t} - r_{f,t-1} )</th>
<th>( r_{f,t-1} - r_{f,t-2} )</th>
<th>( \pi_{t+1} - \pi_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{f,t-1} - r_{f,t-2} )</td>
<td>-0.16 [0.00]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_{t+1} - \pi_t )</td>
<td>0.04 [0.27]</td>
<td>0.01 [0.90]</td>
<td></td>
</tr>
<tr>
<td>( \pi^<em>_{t+1} - \pi^</em>_t )</td>
<td>0.08 [0.04]</td>
<td>-0.01 [0.86]</td>
<td>0.26 [0.00]</td>
</tr>
</tbody>
</table>
Table 2

Joint dynamics of riskless rates, risk premia, and market betas

Panel A shows the results of time series regressions of excess returns of the market portfolio (EMKT) on the riskless rate \( (r_f,t) \), one-period change in the riskless rate \( (\Delta r_f,t) \), inflation \( (\pi_t) \) measured as log change in the US Consumer Price Index (CPI), and the return on the 90-day Treasury yield \( (r_{T90,t}) \). Panel B uses the reduced-form empirical model to estimate time series alpha \((\alpha)\), and conditional market beta parameters \((\beta_m, \beta_{m\Delta}, \text{and} \beta_{mM})\) of long-short portfolios. \( \beta_{m\Delta} \) measures the covariance of market beta with \( \Delta r_f,t \), the change in the riskless rate, and \( \beta_{mM} \) measures the covariance of market beta with \( r_f,t \), the level of the riskless rate. \( \bar{r}_{f,t} = r_{f,t} - \bar{r}_f + \lambda_l \) represents the change in riskless rate (adjusted for its mean to equal 0.27, the estimated premium parameter from Table 4, Panel A), and \( \bar{\Delta} r_f,t = \Delta r_f,t - \bar{r}_{f,t} + \lambda_{\Delta} \) represents the change in the riskless rate (with a mean-adjustment to -0.04, its estimated premium parameter from Table 4). The test portfolios are high market beta minus low market beta \((BETA)\), small minus big \((SMB)\), value minus growth \((HML)\), winners minus losers \((WML)\), high volatility minus low volatility \((VOL)\), high residual variance minus low residual variance \((IVOL)\), robust minus weak profitability \((RMW)\), and conservative minus aggressive investments \((CMA)\). The numbers in alternate rows represent \( t \)-statistics using White robust standard errors. The sample period is from July 1963 to December 2014. CAPM = capital asset pricing model.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Intercept</th>
<th>( r_{f,t} )</th>
<th>( \Delta r_{f,t} )</th>
<th>( \pi_t )</th>
<th>( r_{T90,t+1} )</th>
<th>( \Delta r_{f,t+1} )</th>
<th>( R^2 )</th>
<th>Adj. ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( EMKT_{t+1} )</td>
<td>1.10</td>
<td>-1.44</td>
<td></td>
<td></td>
<td></td>
<td>( \bar{r}_{f,t+1} )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(3.32)</td>
<td>(-1.96)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3.32)</td>
<td>(1.96)</td>
</tr>
<tr>
<td></td>
<td>0.98</td>
<td>-1.17</td>
<td>-8.52</td>
<td></td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(2.98)</td>
<td>(-1.60)</td>
<td>(-2.24)</td>
<td></td>
<td></td>
<td></td>
<td>(2.98)</td>
<td>(-1.60)</td>
</tr>
<tr>
<td></td>
<td>1.03</td>
<td>-0.76</td>
<td>-8.57</td>
<td>-1.49</td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(3.02)</td>
<td>(-1.01)</td>
<td>(-2.25)</td>
<td>(-1.11)</td>
<td></td>
<td></td>
<td>(3.02)</td>
<td>(-1.01)</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>-1.49</td>
<td>-7.80</td>
<td>2.85</td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(3.04)</td>
<td>(-1.87)</td>
<td>(-1.93)</td>
<td>(1.01)</td>
<td></td>
<td></td>
<td>(3.04)</td>
<td>(-1.87)</td>
</tr>
<tr>
<td></td>
<td>1.01</td>
<td>-1.25</td>
<td>-9.01</td>
<td>-3.40</td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(3.01)</td>
<td>(-1.70)</td>
<td>(-2.41)</td>
<td>(-0.86)</td>
<td></td>
<td></td>
<td>(3.01)</td>
<td>(-1.70)</td>
</tr>
</tbody>
</table>
Panel B: Unconditional alphas and beta dynamics of long-short portfolio

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Static CAPM</th>
<th>Unconditional CAPM</th>
<th>ICAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta_{1,m}$</td>
<td>$\beta_{1,ml}$</td>
</tr>
<tr>
<td>$BETA_{t+1}$</td>
<td>-0.30</td>
<td>0.03</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(-1.80)</td>
<td>(0.10)</td>
<td>(17.25)</td>
</tr>
<tr>
<td>$SMB_{t+1}$</td>
<td>0.13</td>
<td>0.09</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(1.12)</td>
<td>(0.57)</td>
<td>(7.20)</td>
</tr>
<tr>
<td>$HML_{t+1}$</td>
<td>0.46</td>
<td>0.06</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(4.11)</td>
<td>(0.36)</td>
<td>(-5.39)</td>
</tr>
<tr>
<td>$WML_{t+1}$</td>
<td>0.75</td>
<td>0.68</td>
<td>-0.13</td>
</tr>
<tr>
<td></td>
<td>(4.55)</td>
<td>(2.60)</td>
<td>(-2.51)</td>
</tr>
<tr>
<td>$VOL_{t+1}$</td>
<td>-0.68</td>
<td>-0.40</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>(-3.44)</td>
<td>(-1.37)</td>
<td>(16.60)</td>
</tr>
<tr>
<td>$IVOL_{t+1}$</td>
<td>-0.68</td>
<td>-0.44</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(-3.52)</td>
<td>(-1.60)</td>
<td>(14.46)</td>
</tr>
<tr>
<td>$RMW_{t+1}$</td>
<td>0.46</td>
<td>0.06</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(4.12)</td>
<td>(0.37)</td>
<td>(-5.37)</td>
</tr>
<tr>
<td>$CMA_{t+1}$</td>
<td>0.30</td>
<td>0.40</td>
<td>-0.11</td>
</tr>
<tr>
<td></td>
<td>(3.45)</td>
<td>(3.26)</td>
<td>(-4.87)</td>
</tr>
</tbody>
</table>

44
Table 3
Cross-sectional tests of the capital asset pricing model (sets of 25 test portfolios)

This table shows the results of Fama and MacBeth regressions results. In the first stage, various beta parameters are estimated using a time series regression of excess returns of each test portfolio on the riskless rate ($r_{f,t}$), one-period change in the riskless rate ($\Delta f_{t}$), the market portfolio return ($EMKT$), the interaction of $EMKT$ and $r_{f,t}$, and the interaction of $EMKT$ and $\Delta f_{t}$. In the second stage, these beta parameters are used to estimate the unconditional risk premium parameters ($\lambda$). Each panel reports results using a distinct set of 25 test portfolios constructed based on $5 \times 5$ independent sorts on market equity and on book-to-market (BTM; Panel A), market beta (Panel B), past returns (Panel C), volatility (Panel D), residual volatility (Panel E), profitability (Panel F), and investments (Panel G). Data for these portfolios are downloaded from Kenneth R. French’s website, where details of how they are constructed can be found. Calculation of $R^2$ and adjusted $R^2$ follow Jagannathan and Wang (1996). The numbers in parentheses represent Shanken-corrected $t$-statistics. The sample period is from July 1963 to December 2014.

<table>
<thead>
<tr>
<th>Intercept</th>
<th>$EMKT$</th>
<th>$\Delta f_{t}$</th>
<th>$r_{f,t}$</th>
<th>$EMKT \times r_{f,t}$</th>
<th>$EMKT \times \Delta f_{t}$</th>
<th>$R^2$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Cross-sectional risk premium estimates (25 size and BTM portfolios)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.15</td>
<td>-0.39</td>
<td>0.07</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.05)</td>
<td>(-0.69)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.33</td>
<td>-0.08</td>
<td>0.37</td>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.32)</td>
<td>(-2.27)</td>
<td>(2.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.96</td>
<td>-0.34</td>
<td>-0.07</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.38)</td>
<td>(-0.80)</td>
<td>(-2.56)</td>
<td>(0.20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Cross-sectional risk premium estimates (25 size and market beta portfolios)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>0.03</td>
<td>0.00</td>
<td>-0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.03)</td>
<td>(0.10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.14</td>
<td>-0.07</td>
<td>0.13</td>
<td>0.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.62)</td>
<td>(-2.05)</td>
<td>(0.80)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.34</td>
<td>0.26</td>
<td>-0.03</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.64)</td>
<td>(0.76)</td>
<td>(-1.20)</td>
<td>(1.12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel C: Cross-sectional risk premium estimates (25 size and momentum portfolios)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.41</td>
<td>-0.65</td>
<td>0.08</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.86)</td>
<td>(-1.43)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.22</td>
<td>-0.16</td>
<td>0.64</td>
<td>0.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.63)</td>
<td>(-3.14)</td>
<td>(4.22)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.21</td>
<td>0.41</td>
<td>-0.07</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.60)</td>
<td>(0.85)</td>
<td>(-2.24)</td>
<td>(4.87)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

45
## Panel D: Cross-sectional risk premium estimates (25 size and variance portfolios)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.12</td>
<td>-0.37</td>
<td>0.14</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>(5.56)</td>
<td>(-1.09)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.10</td>
<td>-0.15</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.50)</td>
<td>(-4.07)</td>
<td>(4.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.10</td>
<td>0.69</td>
<td>-0.15</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>(-0.64)</td>
<td>(2.43)</td>
<td>(-5.27)</td>
<td>(2.61)</td>
<td>(1.39)</td>
</tr>
</tbody>
</table>

## Panel E: Cross-sectional risk premium estimates (25 size and residual variance portfolios)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.24</td>
<td>-0.47</td>
<td>0.17</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.79)</td>
<td>(-1.34)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>-0.14</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.29)</td>
<td>(-4.23)</td>
<td>(5.74)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.41</td>
<td>0.16</td>
<td>-0.11</td>
<td>0.37</td>
<td>0.32</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>(2.20)</td>
<td>(0.52)</td>
<td>(-4.05)</td>
<td>(3.22)</td>
<td>(2.20)</td>
<td>(2.54)</td>
<td></td>
</tr>
</tbody>
</table>

## Panel F: Cross-sectional risk premium estimates (25 size and profitability portfolios)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29</td>
<td>0.38</td>
<td>0.06</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.84)</td>
<td>(0.70)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>-0.06</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.62)</td>
<td>(-1.66)</td>
<td>(0.11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.38</td>
<td>0.26</td>
<td>-0.04</td>
<td>0.20</td>
<td>0.12</td>
<td>0.70</td>
<td>0.73</td>
</tr>
<tr>
<td>(1.55)</td>
<td>(0.66)</td>
<td>(-1.61)</td>
<td>(1.35)</td>
<td>(1.06)</td>
<td>(1.96)</td>
<td></td>
</tr>
</tbody>
</table>

## Panel G: Cross-sectional risk premium estimates (25 size and investment portfolios)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.03</td>
<td>-0.27</td>
<td>0.04</td>
<td>-0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.67)</td>
<td>(-0.59)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.35</td>
<td>-0.06</td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.25)</td>
<td>(-1.51)</td>
<td>(1.30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.05</td>
<td>-0.42</td>
<td>-0.01</td>
<td>-0.12</td>
<td>0.17</td>
<td>-0.68</td>
<td>0.69</td>
</tr>
<tr>
<td>(-3.95)</td>
<td>(-1.03)</td>
<td>(-1.06)</td>
<td>(1.36)</td>
<td>(2.84)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4

Cross-sectional tests of the capital asset pricing model (CAPM) (175 test portfolios)

This table shows the results of Fama and MacBeth regressions (described in Table 3) using the joint set of 175 test portfolios constructed based on $5 \times 5$ independent sorts on market equity and on book-to-market (BTM), market beta, past returns, volatility, residual volatility, profitability, and investments. Data for these portfolios are downloaded from Kenneth R. French’s website, where details of how they are constructed can be found. Calculation of $R^2$ and adjusted $R^2$ follow Jagannathan and Wang (1996). The numbers in parentheses represent Shanken-corrected $t$-statistics. The sample period is from July 1963 to December 2014. Panel A shows the results using the unconditional CAPM (described in Table 3). Panel B shows the results for the Jagannathan and Wang (1996) conditional CAPM and the Lettau and Ludvigson (2001) conditional CAPM. DEF denotes the default spread; HC denotes returns to human capital (measured by labor income growth); and CAY, Lettau and Ludvigson’s (2001) consumption to aggregate wealth ratio. Panel C shows the results for two subsamples, divided around 1979 shift in Federal Reserve monetary policy. Panel D controls for realized inflation ($\pi_t$) that could influence the results due to its correlation with the Treasury bill rate.

<table>
<thead>
<tr>
<th>Panel A: Unconditional CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>1.03</td>
</tr>
<tr>
<td>(5.13)</td>
</tr>
<tr>
<td>0.21</td>
</tr>
<tr>
<td>(0.90)</td>
</tr>
<tr>
<td>0.63</td>
</tr>
<tr>
<td>(3.16)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Alternative conditional CAPM models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>1.1</td>
</tr>
<tr>
<td>(5.54)</td>
</tr>
<tr>
<td>1.07</td>
</tr>
<tr>
<td>(5.95)</td>
</tr>
</tbody>
</table>

| Intercept | $EMKT$ | $HC$ | $CAY$ | $EMKT \ast CAY$ | $HC \ast CAY$ | $R^2$ | $\bar{R}^2$ |
| 1.19 | -0.63 | -0.08 | -6.35 | 0.37 | 0.36 |
| (6.15) | (-2.01) | (-0.16) | (-2.71) | |
| 1.27 | -0.70 | -0.04 | 0.28 | -6.33 | 0.15 | 0.41 | 0.40 |
| (6.68) | (-2.23) | (-0.41) | (0.55) | (-2.68) | (0.70) | |
### Panel C: Subsamples

<table>
<thead>
<tr>
<th>Years</th>
<th>Intercept</th>
<th>EMKT</th>
<th>$\Delta_{f,t}$</th>
<th>$r_{f,t}$</th>
<th>EMKT</th>
<th>EMKT * $\Delta_{f,t}$</th>
<th>$r'_{f,t-1}$</th>
<th>$R^2$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963–1979</td>
<td>0.69</td>
<td>-0.28</td>
<td>-0.03</td>
<td>0.10</td>
<td>0.40</td>
<td>-0.22</td>
<td>0.45</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.03)</td>
<td>(-0.48)</td>
<td>(-1.29)</td>
<td>(2.90)</td>
<td>(2.60)</td>
<td>(-1.66)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980–2014</td>
<td>0.84</td>
<td>-0.10</td>
<td>-0.01</td>
<td>0.26</td>
<td>0.16</td>
<td>-0.03</td>
<td>0.53</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.56)</td>
<td>(-0.25)</td>
<td>(-0.71)</td>
<td>(2.86)</td>
<td>(1.46)</td>
<td>(-0.14)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Panel D: Controls for inflation

<table>
<thead>
<tr>
<th>Intercept</th>
<th>EMKT</th>
<th>$\Delta_{f,t}$</th>
<th>$r_{f,t}$</th>
<th>EMKT</th>
<th>EMKT * $\Delta_{f,t}$</th>
<th>$r'_{f,t-1}$</th>
<th>$\pi_t$</th>
<th>EMKT</th>
<th>EMKT * $\pi_t$</th>
<th>$R^2$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.44</td>
<td>0.15</td>
<td>-0.05</td>
<td>0.25</td>
<td>0.20</td>
<td>0.14</td>
<td>-0.01</td>
<td>0.08</td>
<td>0.67</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2.32)</td>
<td>(0.49)</td>
<td>(-2.36)</td>
<td>(3.58)</td>
<td>(2.27)</td>
<td>(0.93)</td>
<td>(-0.15)</td>
<td>(0.59)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5
Cross-sectional tests of multifactor models

This table shows the results of Fama and MacBeth regressions using the joint set of 175 test portfolios (described in Tables 3 and 4). Panel A tests an intertemporal capital asset pricing model (ICAPM) in which investors hedge changes in the one-month interest rate using the 90-day Treasury bill. It also reports a test in which the contemporaneous change in the interest rate ($\Delta f_{t+1}$) plays the role of a priced state variable in addition to its lagged value proxying for the conditioning information of the representative investor. Panel B reports the results using the Fama and French three- and five-factor models (1993, 2015). SMB denotes their small minus big factor; HML, their high minus low book-to-market factor; RMW, their robust minus weak profitability factor; and CMA, their conservative minus aggressive investments-based factor. Panel C reports the results of a multifactor model in which CMA is priced along with the market portfolio and the 90-day Treasury bill return. Calculation of $R^2$ and adjusted $R^2$ follow Jagannathan and Wang (1996). The numbers in parentheses represent Shanken-corrected $t$-statistics. The sample period is from July 1963 to December 2014.

### Panel A: ICAPM (one state variable: shocks to interest rates)

<table>
<thead>
<tr>
<th>Intercept</th>
<th>EMKT</th>
<th>$\Delta f_t$</th>
<th>$r_{f,t}$</th>
<th>EMKT $\ast \Delta f_t$</th>
<th>EMKT $\ast r_{f,t-1}$</th>
<th>$r_{T90,t+1} - r_{f,t}$</th>
<th>$\Delta f_{t+1}$</th>
<th>$R^2$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>-0.08</td>
<td>0.03</td>
<td>0.09</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.22)</td>
<td>(-0.24)</td>
<td>(1.29)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.37</td>
<td>0.21</td>
<td>-0.04</td>
<td>0.23</td>
<td>0.20</td>
<td>0.33</td>
<td>0.07</td>
<td>0.67</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>(1.83)</td>
<td>(0.62)</td>
<td>(-1.96)</td>
<td>(3.20)</td>
<td>(2.18)</td>
<td>(1.91)</td>
<td>(2.75)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.09</td>
<td>-0.33</td>
<td>0.27</td>
<td>0.20</td>
<td>0.26</td>
<td>-0.02</td>
<td>0.63</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.48)</td>
<td>(-0.94)</td>
<td>(0.89)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.53</td>
<td>0.06</td>
<td>-0.03</td>
<td>0.27</td>
<td>0.20</td>
<td>0.26</td>
<td>-0.02</td>
<td>0.63</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>(2.69)</td>
<td>(0.17)</td>
<td>(-1.87)</td>
<td>(3.73)</td>
<td>(2.18)</td>
<td>(1.60)</td>
<td>(-1.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Panel B: Fama and French factors

<table>
<thead>
<tr>
<th>Intercept</th>
<th>EMKT</th>
<th>SMB</th>
<th>HML</th>
<th>RMW</th>
<th>CMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.11</td>
<td>-0.53</td>
<td>0.16</td>
<td>0.30</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td>(5.82)</td>
<td>(-1.65)</td>
<td>(1.23)</td>
<td>(2.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.47</td>
<td>0.05</td>
<td>0.28</td>
<td>0.08</td>
<td>0.35</td>
<td>0.47</td>
</tr>
<tr>
<td>(2.81)</td>
<td>(0.18)</td>
<td>(2.21)</td>
<td>(0.50)</td>
<td>(3.11)</td>
<td>(4.48)</td>
</tr>
</tbody>
</table>

### Panel C: Arbitrage pricing theory

<table>
<thead>
<tr>
<th>Intercept</th>
<th>EMKT</th>
<th>$\Delta f_t$</th>
<th>$r_{f,t}$</th>
<th>EMKT $\ast \Delta f_t$</th>
<th>EMKT $\ast r_{f,t-1}$</th>
<th>CMA</th>
<th>$r_{T90,t+1} - r_{f,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>-0.25</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.98)</td>
<td>(-0.88)</td>
<td>(0.35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.28</td>
<td>0.30</td>
<td>-0.05</td>
<td>0.22</td>
<td>0.27</td>
<td>-0.06</td>
<td>0.30</td>
<td>0.71</td>
</tr>
<tr>
<td>(1.71)</td>
<td>(1.00)</td>
<td>(-2.54)</td>
<td>(3.30)</td>
<td>(3.10)</td>
<td>(-0.35)</td>
<td>(2.71)</td>
<td></td>
</tr>
<tr>
<td>0.14</td>
<td>0.42</td>
<td>-0.05</td>
<td>0.19</td>
<td>0.26</td>
<td>0.13</td>
<td>0.27</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.72)</td>
<td>(1.25)</td>
<td>(-2.50)</td>
<td>(2.84)</td>
<td>(3.05)</td>
<td>(0.80)</td>
<td>(2.49)</td>
<td>(2.43)</td>
</tr>
</tbody>
</table>

49
Fig. 1. Time series of the annualized one-month Treasury bill rate and its first difference.

Fig 2. Predicted market risk premium. This figure plots the predictions of next period’s market excess return using the level of the one-month Treasury bill rate as its first difference (cyan line), only the one-month Treasury bill yield (blue line), and the average market excess return in the sample period from 1963 to 2014. The grey bars represent National Bureau of Economic Research–designated recessions.
Static CAPM

Unconditional CAPM

Size and book-to-market portfolios

Size and momentum portfolios

Size and profitability portfolios
Size and investment portfolios

Fig. 3. Model implied and estimated risk premia [static and unconditional capital asset pricing model (CAPM)]. Each circle in the graph represents a portfolio, with the realized average return as the vertical axis and the fitted expected return as the horizontal axis. The radius of the circle is proportional to the market capitalization of the portfolio. Green and red represent the ten portfolios in the lowest and highest quintiles in the non-size dimension, such as book-to-market, or momentum. For each portfolio $i$, the realized average return is the time series average of the portfolio return, and the fitted expected return is the fitted value for the expected return using estimates from a cross-sectional regression. The straight line marks 45 degrees from the origin.

Fig. 4. Model implied and estimated risk premia for 175 portfolios [static and unconditional capital asset pricing model (CAPM)]. Blue dots represent predictions of the unconditional CAPM; orange dots, the static CAPM. Each scatter point in the graph represents a portfolio, with the realized average return as the vertical axis and the fitted expected return as the horizontal axis. For each portfolio $i$, the realized average return is the time series average of the portfolio return, and the fitted expected return is the fitted value for the expected return using estimates from a cross-sectional regression. The straight line marks 45 degrees from the origin.