Default probabilities and business cycle regimes:

a forward-looking approach

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Motivation:

- Importance of business cycle effects on risk measurement
- Risk-sensitivity / Procyclicality trade-off in Basel II

Plan of the presentation:

1) Basel II and the procyclicality issue
2) Possible ways to deal with procyclicality
3) Model in Pederzoli and Torricelli (2005)
4) Publicly available Italian default data
Time dimension of credit risk

Dependence on the general economic conditions through systematic risk factors:


In particular:
Bangia et al. (2002): recession and expansion regimes in transition matrices
Procyclicality

Amplification of the business cycle due to the risk-sensitivity of capital requirements

→ reduction in the capital ratio during recession and vice versa, i.e. reduction in lending when the economy is in a downturn

\[ \frac{RC}{\sum_{i} w_i A_i} \geq 8\% \]

Jackson et al. (1999) → procyclicality in Basel I

Many concerns about procyclicality in Basel II, e.g. Danielsson et al. (2001), Borio et al. (2001), Lowe and Segoviano (2002), Kashyap and Stein (2004), Gordy (2004), ...
Business cycle and credit risk models:

Where?

A) Rating Systems:

• Rating Assignment

• Rating Quantification (Probability of Default)

B) Loss Given Default

C) Exposure at Default

D) Correlations


Focusing on (A):

Rating Assignment:

➢ Point in time

➢ Through the cycle

Rating Quantification:

➢ Unconditional PDs

➢ Time-varying / business cycle dependent PDs
Rating systems in current practice

➢ Rating Agencies: through the cycle ratings logic (pessimistic scenario)
➢ Banks: typically point in time logic (based on accounting or market data)

Basel II requires:

• through the cycle ratings;
• PDs estimated as long-run averages;

→ neutralization of the business cycle effects
→ time dimension of risk neglected
Three ways to deal with procyclicality in IRB

1. Smoothing inputs to the capital function

2. Flattening the capital function itself

3. Smoothing the output of the capital function
Including the time dimension of risk → economic conditions over the credit horizon (typically one-year)

Two views of the business cycle:

1) business cycle too irregular to be predicted → current conditions as the best forecast for future conditions

2) business cycle –at least partly- predictable → appropriate forecast over the credit horizon
The model proposed in Pederzoli and Torricelli (2005) aims at:

- including business cycle effects;
- smoothing procyclicality by introducing economic forecasts in the PDs estimation.

→ ttc ratings + time-varying forward-looking PDs

PDs estimation based on:

1. Conditional expansion and recession PDs for each rating class;
2. Recession probability forecasts.
Models for business cycle dependent PDs estimation

- Specific macroeconomic variables (e.g. GDP), continuous values
  → CreditPortfolioView
- Business cycle as discrete variable, typically two values (expansion and recession)
  → Bangia et al. (2002)

**Why a discrete binomial model?**

- Evidence of good representativeness (e.g. Bangia et al. (2002))
- Econometric models for prediction of the business cycle states more accurate and stable (e.g. Estrella et al. (2003))
- From a regulatory point of view less variability in the binary representation
The proposed model

A.1 One-period model: period length equal to the credit horizon $k$, $k \in \mathbb{N}$.

A.2 Business cycle state over $[t, t+k]$ binomial variable:

$$S_{t+k} = \begin{cases} 
E & P(E) \\
R & P(R) 
\end{cases}$$

$S_{t+k} =$ business cycle state over $[t, t+k]$;

$E =$ expansion state;

$R =$ recession state;

$P(E) =$ probability of an expansion over one period;

$P(R)=1-P(E) =$ probability of a recession over one period.

A.3 $P(R)$ conditioned on the information available in $t$, $I_t$

$$P_t(S_{t+k} = R) = P(S_{t+k} = R \mid I_t) = f(\beta' x_t)$$

$x_t =$ vector of explanatory variables for the business cycle regime, $x_t \in \mathbb{R}^n$;

$\beta =$ vector of coefficients.
A.4 Default rate for each rating class stochastic variable with state-dependent distribution:

\[ f(DR_{t+k} \mid S) = \begin{cases} f_E(DR_{t+k}) & \text{if } S = E \\ f_R(DR_{t+k}) & \text{if } S = R \end{cases} \]

\(DR_{t+k}\) = default rate over \([t, t+k]\);

\(f_E\) = probability distribution of the default rate conditional on the state of expansion;

\(f_R\) = probability distribution of the default rate conditional on the state of recession.

**Conditional and unconditional default rate distribution**
→ ex-ante mixture distribution:

\[ f_t(DR_{t+k}) = P_t(S_{t+k} = E) \times f_E(DR_{t+k}) + P_t(S_{t+k} = R) \times f_R(DR_{t+k}) \]

By defining the conditional default probabilities as:

\[ PD_E = E(DR \mid E) \quad PD_R = E(DR \mid R) \]

the ex-ante (unconditional) default probability on \([t, t+k]\) is:

\[ PD_t = E_t(DR_{t+k}) = \int DR_{t+k} f_t(DR_{t+k}) dDR_{t+k} \]
\[ = P_t(S_{t+k} = E) \times PD_E + P_t(S_{t+k} = R) \times PD_R \]

→ analogous to Basel II if \(P(E), P(R)\) long-run sample proportion

→ but \(P(E), P(R)\) forward-looking over the credit horizon
Time inconsistency:

Basel II (and credit risk models in general): \( k = \text{one year} \)

Business cycle chronology \( \rightarrow \) months/quarters

By dividing \([t,t+k]\) in \( n \) sub-periods of length \( k/n \):

\[
S_{t_i} = \begin{cases} 
E & \text{if } [t_{i-1}, t_i] \text{ expansion} \\
R & \text{if } [t_{i-1}, t_i] \text{ recession}
\end{cases} \quad i = 1, \ldots, n
\]

\[
t_i = t + i \times \frac{k}{n} \quad i = 0, \ldots, n
\]

\( k = 12 \text{ months}; \ n = 4 \).

\[
\begin{array}{cccc}
S_{t_1} & S_{t_2} & S_{t_3} & S_{t_4} \\
\hline
\end{array}
\]

\( \rightarrow \) Forecast over each quarters

\( \rightarrow \) Alternatively, proxy consistent with procyclicality target:

forecast of \( S_{t_4} \)
Application to Italian data

United States:
(application in Pederzoli and Torricelli (2005))

• Transition and default data from Standard & Poor’s
  (Bangia et al. (2002))
• NBER business cycle chronology

Italy:
• Quarterly default data from Bank of Italy (1990 Q1-2002 Q4)
• ISAE/ECRI business cycle chronology
Default data from Bank of Italy

(Base Informativa Pubblica):

Quarterly default rates

Default rates calculation:

default rate $DR_{t,t+k}$ for the period $[t,t+k]$:

$$DR_{t,t+k} = \frac{D_{t,t+k}}{PL_t}$$

$D_{t,t+k} = \text{defaults flow over the period } [t,t+k]$

$PL_t = \text{stock of performing loans at time } t$

Quarterly percentage default rates historical series
Expansion and Recession regimes in the default rates

Business Cycle chronology:

- ISAE (*Istituto di Studi e Analisi Economica*)
- ECRI (*Economic Cycle Research Institute*)

ISAE chronology

<table>
<thead>
<tr>
<th>Peaks</th>
<th>Troughs</th>
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<tbody>
<tr>
<td>Oct-70</td>
<td>Oct-71</td>
</tr>
<tr>
<td>Mar-74</td>
<td>May-75</td>
</tr>
<tr>
<td>Feb-77</td>
<td>Dec-77</td>
</tr>
<tr>
<td>Mar-80</td>
<td>Mar-83</td>
</tr>
<tr>
<td>Mar-92</td>
<td>Jul-93</td>
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<tr>
<td>Nov-95</td>
<td>Nov-96</td>
</tr>
<tr>
<td>Dec-00</td>
<td></td>
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<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Expansion</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td># periods</td>
<td>44</td>
<td>35</td>
<td>9</td>
</tr>
<tr>
<td>mean DR</td>
<td>0.00643</td>
<td>0.00622</td>
<td>0.00727</td>
</tr>
<tr>
<td>std DR</td>
<td>0.00129</td>
<td>0.00127</td>
<td>0.00096</td>
</tr>
</tbody>
</table>

neglecting 2001, 2002 (likely structural break)
Proxy for ratings

→ borrower’s geographic area (South, Centre, North)
→ borrower’s dimension (Small Business, Large/Medium Corporates)

Proxy:

➢ global loan size < 500.000 euro → Small business
➢ global loan size > 500.000 euro → Medium/Large corporates

Conditional PD estimates by ‘rating class’

<table>
<thead>
<tr>
<th></th>
<th>Recession</th>
<th>Expansion</th>
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</thead>
<tbody>
<tr>
<td>South Small</td>
<td>0.01212</td>
<td>0.01043</td>
</tr>
<tr>
<td>South Medium-Large</td>
<td>0.01842</td>
<td>0.01551</td>
</tr>
<tr>
<td>Centre Small</td>
<td>0.00763</td>
<td>0.00705</td>
</tr>
<tr>
<td>Centre Medium-Large</td>
<td>0.01174</td>
<td>0.01069</td>
</tr>
<tr>
<td>North Small</td>
<td>0.00480</td>
<td>0.00409</td>
</tr>
<tr>
<td>North Medium-Large</td>
<td>0.00702</td>
<td>0.00546</td>
</tr>
</tbody>
</table>
Recession Probability Forecast

Probit Model:

\[ P_t(R_{t+k} = 1) = \Phi(\beta'X_t) \]

\[ R_t = \begin{cases} 
1 & \text{recession in quarter } t \\
0 & \text{expansion in quarter } t 
\end{cases} \]

\[ X_t = \text{exp lanatory variables} \]

\[ \rightarrow \text{Domestic and international financial variables (Artis et al. (2004))} \]

- Short and long interest rates from Italy, Germany
- Term Spread from Italy, Germany, US
- Equity Indices return from Italy, Germany, US
Sample: 1970 Q1 – 2002 Q4

Selection based on SIC criterion:

→ German term spread

→ Italian long term interest rate
Capital Requirements

Foundation IRB Approach
LGD = 45%, M= 2,5, EAD=100

→ ISAE or ECRI?
Recession probability forecast – ECRI chronology

Basel II Capital Requirements (CR)
Open points

1. Which chronology does better represent Italian default rates dynamics?

2. European business cycle? Relationship with default rates.