Tools and Concepts for the Modern Economist

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1 Background and Goals

This course is a bird's eye introduction to the concepts that underlie most techniques and frameworks used in economics and finance (and also engineering and physics), providing the foundation for understanding no-arbitrage, general equilibrium models, statistical models, and machine learning. The major topics: mathematical concepts, convex optimization, and signal processing are interconnected and feed from each other. They will likely also feed into the student's repertoire and overall comfort with quantitative research. In informal terms we will discuss what it can mean that a model is "close" to reality, or an other model, or how to engineer a decomposition of something observed in reality into fundamental simple parts.

The course assumes only a basic command of linear algebra and ordinary calculus and develops the material from first principles. After having taken this course, the student encountering a problem within the families discussed will be better able to

1. detect its origin
2. assess feasibility
3. apply and develop the available solution methods

Conversely, the material will also enhance a student's ability to formulate problems which are feasible. The course makes many practical examples across different fields. Well-known topics such as linear regression and the capital asset pricing model, the Hansen-Jagannathan bound follow as natural applications of a more general theory, along with more recent and related concepts such as LASSO and ridge regressions.

2 Topics in More Detail

2.1 Fundamentals

We briefly review linear vector spaces and introduce the notions of closeness and distance quantitatively through sequences and series, norms, convergence, and the Cauchy criterion. Next we will introduce additional useful structure in the form of subspaces and hyperplanes, linear combinations and linear varieties, convexity and cones, transformations and continuity. We will then introduce l_p and L_p spaces, Banach spaces, as well as the notions of denseness and separability. Dual spaces will be an important concept for optimization. The structure that comes with Hilbert Spaces, for which we introduce inner products, orthogonality, and projections, is an indispensable tool in particular for signal processing introduced below in Section 2.2. We will understand from first principles elementary mathematics workhorses such as the Cauchy-Schwartz inequality.

No-arbitrage itself is a prime application of the ideas developed in this section, along with important concepts such as the CAPM and the Hansen and Jagannathan (1991) framework.

2.2 Signal Processing

This section starts from the basic construction of Fourier series, and subsequently the Fourier transform, with their strong ties to L_2 and L_1 spaces introduced in Section 2.1. We will discuss tempered distributions along with generalized derivatives, convolution, the central limit theorem as one of its applications, and filtering. There are many concrete examples in the field, such as Chen and Joslin (2012), Filipović et al. (2013), Dew-Becker and Giglio (2016), Schneider (2015), and Schneider (2018).

A big part of this section concerns practical application and the discrete Fourier transform. The class will close, time allowing, with an introduction to the Radon transform, a technique used in computer tomography, amply applicable, yet entirely unexplored in finance.

2.3 Convex Optimization

The collection of topics in this chapter use all concepts introduced in Section 2.1 above, in particular convex sets and cones, halfspaces and hyperplanes, as well as many introduced in Section 2.2. These concepts are useful for (generalized) inequalities and geometric optimization. We will also discuss conjugate functions, Fenchel's inequality, log-concave and log-convex functions and subsequently the Lagrange dual problem. Virtually any economic model is an instance of the so-called generalized moment problem, within which we will feature applications like Ross (2015) and Schneider and Trojani (2017).

3 Literature

We will use textbooks for this class, in particular Luenberger (1997), Boyd and Vandenberghe (2004), and Osgood (2007), as well as papers which will be introduced in the slides.

4 Grading

There will be a take-home exam in the form of a discussion of research papers in light of the concepts developed in the class, as well as computation exercises.

References

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